Multiobjective Reactive Power Compensation with an Ant Colony Optimization Algorithm

P. Gardel, B Barán, H Estigarribia, U Fernández, S. Duarte

National University of Asuncion, Paraguay

{pgardel, bbaran, hestigarribia} @cnc.una.py, ufernandez@fulbrigthtweb.org, Susana2.Duarteflores@uwe.ac.uk.

Keywords: Reactive Power Compensation, Multiobjective Optimization, Ant Colony Optimization.

Abstract

This paper presents an Ant Colony Optimization (ACO) algorithm applied to the reactive power compensation problem in a multiobjective context. The developed algorithm was denominated *Electric Omicron* (EO) given that it was inspired in the *Omicron ACO* proposed by some of the authors. The proposed EO algorithm was compared to a variant of the SPEA (*Strength Pareto Evolutionary Algorithm*), specially designed for this problem. This variant of SPEA has previously shown an excellent performance in this type of problem. Experimental results presented in this paper show that the proposed EO outperforms SPEA, i.e., EO finds better Pareto solutions considering voltage deviation and investment. As long as we know, this is the first attempt to solve the reactive power compensation problem with an ACO algorithm in a multiobjective context.

1 Introduction

In an electric power system, the main goal related to the reactive power compensation problem is to determine the adequate size and the physical distribution of capacitive or inductive banks. Traditionally, this problem is addressed as a single objective optimization problem (SOP) [3,4,8]. A Single-objective Optimization Algorithms (SOA) usually provides a unique optimal solution. Typically, the objective function is formulated as a lineal combination of several factors such as investment or transmission losses, that are subject to operational constrains such as reliability and voltage profile [8]. These factors that are considered as the optimization objectives usually are contradictory, making very difficult to find the right lineal combination. Considering this situation, Multiobjective Optimization Algorithms (MOA) were proposed to optimize independent and simultaneously several objectives. Therefore, a MOA usually provides a whole set of optimal tradeoff solutions known as *Pareto set*. The Pareto set gives the engineer the opportunity to consider more options before making a final decision.

The new approach introduced in this paper to solve the reactive power compensation problem is based on the *Omicron* ACO (OA), proposed by Barán et al. [6, 7], a variant of the Ant Colony Optimization algorithm (ACO) that has proved to be a very competitive algorithm solving several optimization problems [5].

2 Multi-objective Optimization Problem

A general Multiobjective Optimization Problem [11] includes a set of *k* objective functions and *m* restrictions that are functions of a set of *n* decision variables. This can be expressed as:

Optimize	$y = \mathbf{F}(\mathbf{x}) = [F_1(\mathbf{x})\ F_2(\mathbf{x})\ \ldots F_k(\mathbf{x})]$,
Subject to	$e(\mathbf{x}) = [e_1(\mathbf{x})\ e_2(\mathbf{x})\ \ldots\ e_m(\mathbf{x})] \le 0$,
where	$\mathbf{x} = [x_1\ x_2\ \ldots\ x_n] \in X$,
and	$\mathbf{y} = [y_1\ y_2\ \ldots y_k] \in Y$,

where **x** is the decision vector and **y** the objective vector.

According to this definition, each solution consists of a vector **x**, that yields an objective vector **y.** The restrictions $e(\mathbf{x}) \le 0$ determinate the set of feasible solutions $X_f \subset \mathbf{X}$.

In a multi-objective context, the concept of Pareto dominance [10] is used to compare two solutions. Thus, considering two decision vectors **u** and **v**, in a minimization context, we say that **u** dominates **v,** denoted as $\mathbf{u} \succ \mathbf{v}$, iff:

1- $F_i(u) \le F_i(v), \forall i \in \{1,2,\ldots\ldots,k\}$.

2- $F_i(u) < F_i(v)$ for at least one *i*.

The definition of dominance in a maximization context is formulated in analogous way, just changing the $\le$ and $<$ symbols by $\ge$ and $>$.

The decision vectors **v** and **u,** are said to be non-comparable solutions, denoted by $\mathbf{v} \sim \mathbf{u}$, if neither **v** dominates **v** nor **u** dominates **v,** i.e. **v** is not better or worse than **u,** considering all the objectives at the same time.

The set of feasible solutions that are not dominated is known as the Pareto optimal set which can be formally defined as:

$$P = \{x \in X_f \mid x \succ v \vee x \sim v \quad \forall v \in X_f\}$$

The optimal solution of a MOP is a Pareto set *P*. This is due to the fact that MOP usually does not have a single optimal solution that dominates all other solutions, but a set of tradeoff solutions where neither of them can be considered better or worse than the others.

3 Mathematical formulation of the problem

For the purposes of this paper, the following assumptions where considered in the formulation of the problem:

- The cost per MVAr of the reactive power compensation device is the same for all buses of the system.
- The power system is considered only at peak load.
- Only discrete values are allowed for reactive banks.

- The maximum sizes of reactive and capacitive banks are imposed.

Based on these assumptions, two objective functions were selected to be minimized: investment related to the reactive power compensation device and average voltage deviation.

3.1 Objective functions

The two selected objective functions (F_1, F_2) can be formulated as follows:

F_1: Invest in reactive power compensation device:

$$F_1 = \sum_{i=1}^{n} |B_i| \quad s.t. \quad \begin{cases} 0 \le F_1 \le F_{1m} \\ 0 \le B_i \le B_m \end{cases} \qquad (1)$$

where: F_1 is the total required invest; F_{1m} is the maximum amount available for investment; Bi is the compensation at busbar i, measured in MVAr; B_m is the absolute value, in MVAr, of the maximum amount of compensation allowed at a single busbar of the system and n is the number of busbar in the electric power system.

F_2: Average voltage deviation:

$$F_2 = \frac{\sum_{i=1}^{n} |V_i^* - V_i|}{n} \qquad (2)$$

where: F_2 is the per unit (pu) average voltage difference; V_i is the actual voltage at busbar i (pu) and V_i* is the desired voltage at busbar i.

3.2 Description of the solution representation and the search space.

A solution of the problem is a distribution of the reactive power compensation devices that satisfies the economic and operative restrictions. This solution is represented by a vector $\mathbf{x} = \mathbf{B}$ of dimension n where each entrance b_i associates a value of reactive compensation to busbar i.

For example, considering a three busbar system, $\mathbf{x} = [-2, 0, 3]$ represents a solution where the first busbar has a reactor of 2 MVAr, the second busbar is not compensated and the third busbar has a capacitor of 3 MVAr. Then, for a solution $\mathbf{x}$, the load distribution and the load flow equations determine an objective vector $\mathbf{y} = [F_1(\mathbf{x}), F_2(\mathbf{x})]$.

In real life, the commercial availability of reactive power compensation devices limit the number of possible compensation values that can be considered for each busbar. This number of possible compensations available for a busbar is denoted by c. Thus, a vector of c possible compensation levels may be defined for each busbar i as $\mathbf{B_i} = [b_{i1},\ldots b_{ij},\ldots,b_{ic}]$. For the sake of simplicity, the same value of c was adopted for every busbar.

4 SPEA description.

The algorithm used for comparison was originally proposed by Barán et al. [1] and later improved in [2]. It was inspired in the already well-established SPEA [11]. Several modifications were introduced to the original SPEA algorithm to improve its performance in the reactive power compensation problem, as: a heuristic initialization; a local optimization heuristic technique; two external populations to store the solutions and a technique inspired in Simulated Annealing, known as Freezing. The method proposed by Barán et al. [1,2,9] may be summarized in the following steps:

1. Generation of an initial population *Pop*, using the heuristic method exposed in [1,2,9] and the creation of two empty external nondominated sets P_{known} and SP_{known} (stored external populations)
2. Copy the nondominated members of *Pop* to P_{known} and SP_{known}.
3. Remove individuals within SP_{known}, which are dominated by any member of SP_{known}.
4. Remove solutions within P_{known}, which are dominated by any member of SP_{known}.
5. If the number of solutions in P_{known} exceeds a given maximum g, clustering is applied in order to reduce the external population to a size g.
6. Calculate the fitness of each individual in *Pop* as well as in P_{known} using standard SPEA fitness assignment procedure.
7. Select individuals from *Pop* + P_{known} (multiset union) until the mating pool is filled.
8. Apply the probabilities P_{to} (using the local optimization), P_c (crossover) and P_m (mutation) to determinate whether an individual is locally optimized or selected for crossover and mutation, in which case, standard genetic operators are applied.
9. In case the stop criterion is not verified go to step 2**.**

5 Proposed method: *Electric Omicron* (EO).

Ant Colony Optimization Algorithms (ACO) are inspired in the behavior of real ant colonies [5]. Real (biological) ants communicate to each other in an indirect way using a chemical substance called pheromone. In a similar way, artificial ants, created by an ACO algorithm, communicate to other artificial ants using a matrix $\tau = \{\tau_{ij}\}$, called pheromone matrix. The pheromone matrix summarizes in some way the information already found by former ants guiding new ants to construct potentially good solutions. ACO algorithms also take advantage of heuristic information η called visibility. The visibility used in this paper was especially defined for the reactive power compensation problem and it is explained in subsection 5.1.

An artificial ant uses the information saved in τ and the visibility η to construct potentially good solutions, traveling around all busbars of the power system. At each busbar i an ant determine a probability of selecting a compensation b_{ij} for that busbar i using the following equation:

$$\Pr_{ij} = \begin{cases} \dfrac{\tau_{ij}^{\alpha} \times \eta_{ij}^{\beta}}{\sum \tau_{ij}^{\alpha} \times \eta_{ij}^{\beta}} \end{cases} \qquad (3)$$

where α and β define the relative influence of the heuristic information and the pheromone level.

Once the probability for every b_{ij} has been determined, a probabilistic selection method, like a roulette wheel [2], is used to choose a specific value considering the probabilities associated which each compensation value. The same procedure is applied to determinate the compensation at every busbar of the power system.

All the nondominated solutions are saved in a set known as population *Pop*. Every time a new solution is generated, it is compared to the ones in *Pop*. If the new solution is nondominated witch respect to *Pop* it is kept, otherwise it is dismissed.

This iterative process is repeated *K* times. Then τ is updated witch the *m* solutions saved in *Pop*. The parameter *O* (*Omicron*) defines the amount of pheromone that each solution deposits.

The pheromone matrix associates a row vector to each busbar and a column to every possible compensation b_j. In the updating process, all entrances in τ are firstly settled to an initial value τ_0. In order to update τ, an artificial ant follows the following steps:

1- Take a solution of *Pop*.
2- Check the value b_{ij} of the compensation device settled in busbar *i*.
3- Deposit an amount of *O*/*m* pheromone at the column of τ associated to b_j in the row *i* of τ.
4- Repeat steps 2 and 3 for every busbar in the system.
5- Repeat steps 1 to 4 for every solution in *Pop*.

As a consequence, the entrances in τ only take values between τ_0 and $O + \tau$. The proposed EO algorithm continues this process until a stop condition is reached.

5.1 Visibility

In a single objective context the visibility usually guides the ant to make what seems to be the locally best choice. But, in a multi-objective context is not always possible to determinate the locally best choice because the different objectives may be contradictory among them. So, for the present work, to reduce the voltage deviation the power compensation should be increase and, for other side, to minimize the invest, the power compensation should decrease. Clearly, there is not a unique locally best choice that satisfies both objectives.

Based on this kind of tradeoff considerations, two different functions were created and combined to define the visibility.

Both functions "guide" the ants assigning probabilities to every possible compensation, b_j, at each busbar.

To determinate visibility, a load flow solution is first calculated considering the network with a base compensation distribution, for this paper the visibility is calculated considering the system with no reactive compensation at all. Based on this load flow results, visibility η is defined as a linear combination of two functions: η_1 and η_2.

The first function, η_1, guides the ants to construct good solutions with high compensation values; therefore, with a low voltage deviation. For this purpose, the function η_1 creates a matrix, similar to τ, that associates a row to every busbar and a column to every possible compensation b_j. This matrix assign a probability of been selected to every considered b_j for each busbar. The assignment of probability is made according to the compensation needed in each busbar. So, if inductive compensation is needed the probability grows giving low levels of probabilities of being selected to the capacitive banks and high levels to the capacitive banks (see Fig. 1). The maximum probability assigned to the largest capacitive (or inductive) bank is directly proportional to the voltage deviation in each busbar.

The second function, η_2, guides the ants to construct solutions with low level of compensation, that is, η_2 prioritizes the minimization of investment over the voltage deviation. To achieve its goal, η_2 assigns a low probability of being selected to the banks of great value, capacitive or inductive, and a high probability to small banks and to the null compensation, as shown in Fig. 2.

Finally, η_1 and η_2 are linearly combined to define visibility:

$$\eta[ij] = w_1 * \eta_1[ij] + w_2 * \eta_2[j] \quad (4)$$

where w_1 and w_2 are weight factors (with $\boldsymbol{w_1} + \boldsymbol{w_2} = 1$) that change with the iterations of the algorithm, i.e., $\boldsymbol{w_1}$ is initialized in 1 and $\boldsymbol{w_2}$ in 0; then, $\boldsymbol{w_1}$ dynamically decreases with the number of iterations until it reaches a final value of 0, when $\boldsymbol{w_2}$ reaches a value of 1.

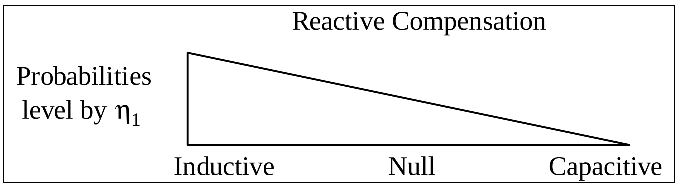

Figure 1: Graphic representation of probabilities assigned by η_1, in this case inductive compensation is needed.

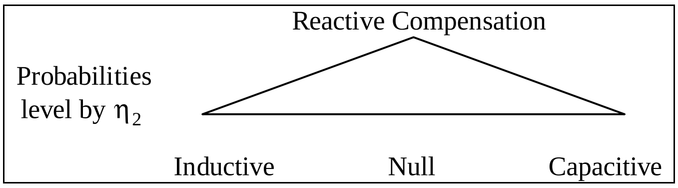

Figure 2: Graphic representation of probabilities assigned by η_2, where null compensation has higher probability.

At the beginning of a run, η assigns probability levels very similar to the figure 1, while at the end, the probability distribution looks like figure 2. At an intermediate iteration with $\boldsymbol{w_1} = \boldsymbol{w_2} = 0.5$, the distribution of probabilities would look like the one shown in figure 3.

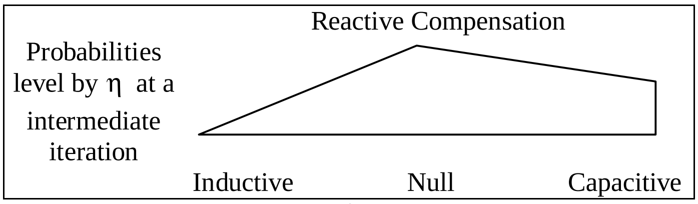

Figure 3: Graphic representation of probabilities assigned by η at an intermediate iteration of generation.

With the utilization of this combined visibility that is changing dynamically we aim to achieve the construction of solutions with high compensation at the beginning of a generation and solutions with low compensation level at the final of the generation.

5.2 Electric Omicron (EO) pseudocode.

A brief pseudocode corresponding to the proposed Electric Omicron (EO) may be expressed as follows:

Read load network parameters.
Randomly, generate initial population
Calculate η_1 and η_2 (visibility functions).
While end condition is not reached
    Update pheromone matrix τ
    For k=1 to K
        Calculate visibility η .
        Ant creates a new solution S_{new}.
        Evaluate S_{new}. (Calculate objective functions)
            If S_{new} is non-dominated (it is a good solution)
                $POP = POP + S_{new}$
            else
                Discard S_{new}.
            end if
    end for
    Eliminate dominated solution from *POP*.
end while.

6 Experimental Results.

In order to compare the proposed EO algorithm to the already established SPEA of Barán et al. [1,2], ten runs of each algorithm were performed. A unique initial population was created and utilized for all runs of the algorithms. This initial population was generated using the heuristic method proposed in [1,2].

The well-known IEEE 118 power system [1,2] is used as a test problem for this paper to facilitated comparison.

For the SPEA algorithm, two sets of five runs each were made. For the first set, 120 generations and 100 individuals were determined, following the suggestions of Barán et al [1,2]. This first set of executions took about 8 hours each. The second set was determined to take twice as long, i.e. it took about 16 hours, and evaluated 230 generations.

In a similar way, two sets of five runs were made with the EO algorithm, taking about 8 hours and 16 hours respectively. The EO algorithm created and evaluated 10,000 solutions in the 8 hours executions set and 20,000 solutions for the sixteen hours executions set. The parameters for the proposed heuristic were arbitrated and settled K = 1000 and O = 1500.

Thus, the following Pareto sets were calculated:

S_1 for 5 runs of the SPEA, in about 8 hours
S_2 for 5 runs of the SPEA, in about 16 hours
S_f for the SPEA, combining S_1 and S_2
EO_1 for 5 runs of the EO, in about 8 hours
EO_2 for 5 runs of the EO, in about 16 hours
EO_f for the EO, combining EO_1 and EO_2

To compare experimental results, the concept of *coverage*, $\mathbb{C}$, is used [2,11]. Given two set of solution, C_1 and C_2, the *coverage* of C_1 over C_2 can be defined as fallows:

$$\mathbb{C}(C_1,C_2)=\frac{\left|\left\{y''\in C_2 ; \exists\ y'\in C_1\ s.t.\, y' \succ y''\right\}\right|}{\left|C_2\right|}*100 \qquad (5)$$

The *coverage* value, $\mathbb{C}(C_1,C_2)$, indicates the percentage of solution of the second set that is dominated by at least one solution of the first set.

In Table 1 the *coverage* values of SPEA over EO are shown. It indicates that almost none solution of a set calculated using EO is dominated by any solution found by the SPEA algorithm. At the same time, the *coverage* values of OE over SPEA shown in table 2 indicate that most solutions calculated with SPEA are dominated by at least one solution found by the EO.

Clearly, Tables 1 and 2 shows that EO solutions are better than the ones calculated with SPEA, most of the time. In fact, no solution of set EO_2 is dominated by any solution calculated with SPEA (see second row of Table 1). On the contrary, every solution of S_1 is dominated by at least one solution of any set calculated with the proposed EO algorithm.

$\mathbb{C}$(***S*, *EO***)	S_1	S_2	S_f
EO_1	0%	1,2%	1,2%
EO_2	0%	0%	0%
EO_f	0%	0%	0%

Table 1: Coverage of SPEA over EO, $\mathbb{C}$(***S*, *EO***).

$\mathbb{C}$(***EO*, *S***)	EO_1	EO_2	EO_f
S_1	100%	100%	100%
S_2	98,2%	100%	100%
S_f	98,2%	100%	100%

Table 2: Coverage value of EO over SPEA, $\mathbb{C}$(***EO*, *S***).

Figure 4 compares the Reactive Power Compensation measured in MVA vs. the Average Voltage in pu, for a run of eight hours of EO and SPEA. Figure 5 shows the same parameters for the sixteen hours execution.

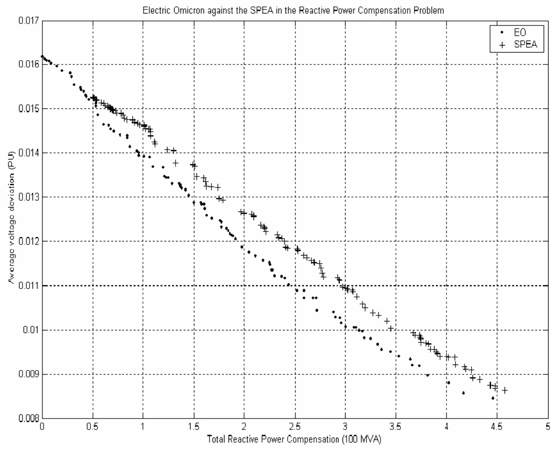

Figure 4: Total reactive power compensation (X-axis) vs. average voltage deviation (Y-axis) for EO and SPEA, after 8 run hours.

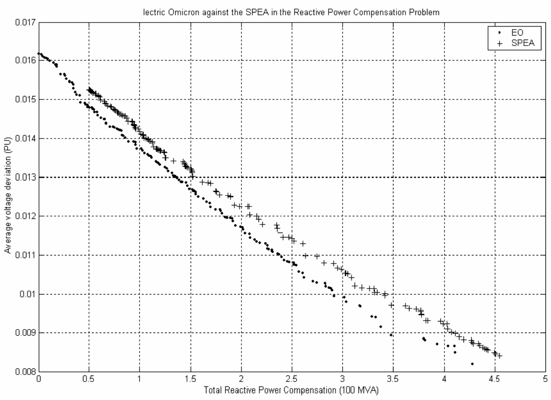

Figure 5: Total reactive power compensation (X-axis) vs. average voltage deviation (Y-axis) for EO and SPEA after 16 run hours.

Clearly, EO outperforms the SPEA algorithm finding better solutions, i.e. EO finds solutions with lower investment than the solutions found by the SPEA algorithm, for similar average voltage deviation.

Conclusion and future work.

This paper presents the *Electric Omicron* algorithm, a specialized version of the *Omicron* ACO to solve the reactive power compensation problem. As long as we know, this is the first time an Ant Colony Optimization algorithm is used to solve this problem in a multiobjective context.

In order to apply the *Omicron* ACO to the reactive power compensation problem a new visibility function was specially designed, combining dynamically two different probability distribution with the aim of obtaining a better set of Pareto solutions in only one run of the EO. Each probability distribution is chosen to improve one of the conflicting objective functions: investment in reactive devices and average voltage deviation.

Analyzing the experimental results we concluded that the *Electric Omicron* is a novel promising alternative to solve reactive power compensation problem. The proposed EO algorithm clearly presents a better performance than the specialized SPEA, one of the most competitive algorithms for this problem [2]. As a matter of fact, the solutions found by the EO algorithm completely dominate the solutions found by the SPEA for the test problem (IEEE-118). This is even more worthy considering that these results have been accomplished by the EO algorithm without any refinement or local search algorithm like the ones used with the SPEA algorithm.

In future works the EO algorithm will be tested with more objective functions as power lose and maximum voltage deviation, for other test problems of different complexity and the value of possible compensation consider for each busbar will be limited to included only commercial compensation devices.

At the same time, the EO algorithm may be modified to include several pheromone matrixes; with this we expect to achieve better Pareto sets with more objectives. Finally, we foresee other interesting applications of ACO algorithms after it becomes better known to electrical engineers, given the excellent experimental results that have already been reported in the specialized literature.

References

[1] Barán B., Vallejos J., Ramos R. and Fernández U., "Multi-Objective Reactive Power Compensation" in *Proc. IEEE Transmission and Distribution Conference and Exposition*, Atlanta, USA. 2001.

[2] Barán B., Vallejos J., Ramos R. and Fernández U., "Reactive Power Compensation using a Multi-Objective Evolutionary Algorithm" in *Proc. IEEE Porto PowerTech'2001.* Porto, Portugal. 2001.

[3] Carlisle J., El-Keib A., Boyd D. and Nolan K., "A Review of Capacitor Placement Techniques on Distribution Feeders" in Proc. *IEEE 29º Southeastern Symposium on System Theory* (SSST'97). 1997.

[4] Delfanti M., Granelli G., Marannino P. and Montagna M., "Optimal Capacitor Placement Using Deterministic and Genetic Algorithms" *IEEE Trans. Power System*, vol. 15, nº3, pp. 1041-1046, August 2000.

[5] Dorigo M. and Di Caro G., "The Ant Colony Optimization Meta-heuristic" Ei David Corne, Marco Dorigo and Fred Glover, editors, *New Ideas in Optimization*, pages 11-32. McGraw-Hill, London, 1999.

[6] Gomez O. and Barán B., "Arguments for ACO's Success", Osvaldo Gomez and Benjamín Barán. Lecture Notes in Computer Science (LNCS). Springer-Verlag. *Genetic and Evolutionary Computing Conference* (GECCO 2004). Seattle, United States. 2004.

[7] Gomez O. and Barán B., **"**Reasons of ACO's Success in TSP" Lecture Notes in Computer Science (LNCS). Springer-Verlag. ANTS 2004 Conference. Brussels, Belgium. 2004.

[8] Miu K., Chiang H. and Darling G., "Capacitor Placement, Replacement and Control in Large-Scale Distribution Systems by a A-Based Two-Stage Algorithms" *IEEE Trans. Powers Systems*, vol. 12, nº3, pp. 1160-1166, August 1997.

[9] Vallejos J., Ramos R. and Barán B., "Multi-Objective Optimization in Reactive Power Compensation" Jornadas de Informática y Telecomunicaciones - Conferencia de Informática y Tecnología Aplicada (JIT-CITA 2001). Asuncion, Paraguay. 2001.

[10] Van Veldhuinzen D., "Multiobjective Evolutionary Algorithms: Classifications, Analysis and New Innovations" Dissertation Ph.D., Faculty of the Graduate School of Engineering, Air Force Institute of Technology, 1997.

[11] Zitzler E. and Thiele L., "Multi-Objective Evolutionary Algorithms: A comparative Case Study and the Strength Pareto Approach", *IEEE Trans. Evolutionary Computation*, vol 3, nº 4, pp. 257-271, November 1999.