Multi-Objective Multicast Routing based on Ant Colony Optimization

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Abstract. This work presents a new multiobjective algorithm based on ant colonies, which is used in the construction of the multicast tree for data transmission in a computer network. The proposed algorithm simultaneously optimizes cost of the multicast tree, average delay and maximum end-to-end delay. In this way, a set of optimal solutions, know as Pareto set, is calculated in only one run of the algorithm, without a priori restrictions. The proposed algorithm was inspired in a Multi-objective Ant Colony System (MOACS). Experimental results prove the proposed algorithm outperforms a recently published Multiobjective Multicast Algorithm (MMA), specially designed for solving the multicast routing problem.

Keywords: Evolutionary Algorithms, Traffic Engineering, Multicast Routing, Multi-objective Optimization, Pareto Front and Ant Colony Optimization.

1. Introduction

Multicast consists of simultaneous data transmission from a source node to a subset of destination nodes in a computer network. Multicast routing algorithms have recently received great attention due to the increased use of new point to multipoint applications, such as radio and TV transmission, on-demand video and teleconferences. Such applications generally have some quality-of-service (QoS) parameters as maximum end-to-end delay and minimum bandwidth resources. Another important consideration in Traffic Engineering is the cost of the tree, understanding cost as other parameters to be minimized, such as: hop count, bandwidth utilization, and others. In this is way; the *Multicast Traffic Engineering Problem* should be treated as a Multi-Objective Problem (*MOP*) [13].

Ant Colony Optimization (*ACO*) is a meta-heuristic proposed by Dorigo et al. [4] inspired by the behavior of ant colonies. In the last few years, *ACO* has empirically shown its effectiveness in the resolution of several different NP-hard combinatorial optimization problems. *ACO* uses a colony of artificial ants, i.e. a set of simple agents that work in a cooperative way and communicate by means of artificial pheromone in the search of better solutions. Several algorithms based on the *ACO* approach consider the multicast routing problem as a mono-objective problem, minimizing the cost of the tree under multiple constrains. In [8] Y. Liu and J. Wu propose the construction of a multicast tree, where only the cost of a tree is minimized. On the other hand, Gu et al. consider multiple parameters of Quality of Service as constrains while minimizing the cost of the tree [7]. These algorithms treat the Traffic Engineering Multicast problem as a mono-objective problem with several constrains. The main disadvantage of this approach is the necessity of an a priori predefined upper bound that can exclude good trees from the final solution.

This work proposes for the first time to solve the Traffic Engineering Multicast problem using the Multi-Objective Ant Colony System (*MOACS*), introduced in [9]. This algorithm optimizes several objectives simultaneously. Experimental results have recently demonstrated that *MOACS* is the best multi-objective *ACO* algorithm for the bi-objective Traveling Salesman Problem (*TSP*) [6].

Besides, to verify the results obtained with the proposed algorithm, it is compared to a Multi-objective Multicast Algorithm (*MMA*) [3]. *MMA* is based on the Strength Pareto Evolutionary Algorithm (*SPEA*) and it simultaneously minimizes three objectives functions for the static case in [1], while in [2] optimizes four objectives for the dynamic case. In summary, this work takes one the finest ant colony multi-objective algorithms, adapting it to the Traffic Engineering Multicast problem.

2. Problem Formulation

For this work, a network is modeled as a direct graph $G=(V, E)$, where V is the set of nodes and E is the set of links. Let:

$(i,j) \in E$: Link from node i to node j; where $i, j \in V$.

$c_{ij} \in \Re^+$: Capacity of link (i, j).

$d_{ij} \in \Re^+$: Delay of link (i, j).

$s \in V$: Source node of a multicast group.

$N_r \subseteq V\text{-}\{s\}$: Set of destinations of a multicast group.

$\phi \in \Re^+$: Traffic demand, in bps.

$T(s,N_r)$: Multicast tree with source in s and a set of destinations N_r.

$p_T(s, n) \subseteq T(s,N_r)$: Path connecting a source node s with a destination node $n \in N_r$.

$d(p_T(s, n))$: Delay of path $p_T(s,n)$, given by the sum of the delays of the path, i.e.:

$$d\left(p_T\left(s,n\right)\right) = \sum_{(i,j)\in p_T(s,n)} d_{ij} \qquad (1)$$

Using the above definitions, a multicast routing problem may be stated as a MOP [13] that tries to find the multicast tree $T(s,N)$[1] that simultaneously minimizes the following objectives:

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[1] For the rest of this work $T \equiv T(s,N_r)$ for further simplicity.

a. Cost of the tree: $$f_1(T) = \phi \cdot \sum_{(i,j)\in T} c_{ij} \quad (2)$$

b. Maximum end-to-end delay: $$f_2(T) = \underset{n\in N_r}{Max} \left\{ d\left(p_T(s,n)\right) \right\} \quad (3)$$

c. Average delay: $$f_3(T) = \frac{1}{|N_r|} \cdot \sum_{n\in N_r} d\left(p_T(s,n)\right) \quad (4)$$

Considering two solutions T and T', for the same multicast group (s,N_r):

$x = [f_1(T) \quad f_2(T) \quad f_3(T)]$ and $z = [f_1(T') \quad f_2(T') \quad f_3(T')]$, only one of the following three conditions can be given:

$x \quad z$ (x dominates z) iff $x_i \le z_i \wedge x_i \ne z_i \; \forall i \in \{1,2,3\}$

$z \quad x$ (z dominates x) iff $z_i \le x_i \wedge z_i \ne x_i \; \forall i \in \{1,2,3\}$ (5)

$x \sim z$ (x and z are non-comparable) iff $x_i \quad z_i \wedge z_i \quad x_i \; \forall i \in \{1,2,3\}$

Alternatively, for the rest of this work, $x \quad z$ will denote that $x \quad z$ or $x \sim z$. A decision vector T is non-dominated with respect to a set Q iff: $T \quad T'$, $T' \in Q$. When T is non-dominated with respect to the whole domain of feasible solutions, it is called an optimal Pareto solution; therefore, the *Pareto optimal set* X_{true} may be formally defined as:

$X_{true} = \{T \in X_f \mid T$ is non-dominated with respect to $X_f\}$ (6)

The corresponding set of objectives $Y_{true}=f(X_{true})$ constitutes the *Optimal Pareto Front.*

3. Multi-objective Ant Colony Optimization algorithm

The Multi-objective Ant Colony Optimization algorithm *(MOACS)*, proposed in [9], is a generalization of the Ant Colony System (*ACS*) [5]. This approach uses a colony of ants for the construction of *m* solutions T at every generation. Then, the known Pareto Front Y_{know} [13] is updated, including all non-dominate solutions. Finally, the pheromone matrix τ_{ij} is updated. Figure 1 presents a *MOACS* general procedure.

```
Read multicast group (s, N_r) and traffic demand φ
Initialize τ_ij
while stop criterion is not verified
    repeat for k=1 to m
        T = Build Tree (Algorithm 3)
        if (T    {T_x | T_x ∈ Y_know}) then
            Y_know = Y_know ∪ T − {T_y | T   T_y} ∀T_y ∈ Y_know
        end if
    end repeat
    Update of τ_ij
end while
```

Figure 1. General Procedure of *MOACS (Algorithm 1)*

The update of pheromone matrix τ_{ij} depends on the state of Y_{know}. If Y_{know} was modified, then τ_{ij} is re-initialized ($\tau_{ij}=\tau_0$) to improve exploration; otherwise, a global update of τ_{ij} is made using the solutions of Y_{know} for a better exploitation, as shown in. Figure 2.

```
repeat for every T ∈ Y_know
    repeat for every (i, j) ∈ T
        τ_ij =(1-ρ).t_0 +ρ.Δt
    end repeat
end repeat
```

Figure 2. Global Update of τ_{ij} *(Algorithm 2)*

with:

$$\Delta\tau = \frac{1}{\sum_{\forall T \in Y_{know}} (f_1(T) + f_2(T) + f_3(T))} \tag{7}$$

where:

- $f_1(T)$ Normalized cost of T, given by equation (2).
- $f_2(T)$ Normalized average delay of T, given by equation (3).
- $f_3(T)$ Normalized maximum end-to-end delay of T, given by equation (4).
- $\rho \in (0,1]$ Trail persistence.

An ant begins the construction of a solution in the source s. A non-visited node is pseudo-randomly [9] selected at each step. This process continues until all desired destinations are reached. Consider N as the list of possible starting nodes, N_i as the list of feasible neighboring nodes to node i, D_r as the set of destinations already reached and φ as another trail persistence parameter. Figure 3 shows the procedure to find a solution T.

```
Initialize T, N and D_r
Repeat until (N = ∅ o D_r = N_r)
    Select node i of N and build set N_i
    if (N_i = ∅) then
        N = N – i                /* erase node without feasible neighbor */
    else
        Select node j of N_i     /*pseudo-random rule */
        T = T ∪ (i, j)
        N = N ∪ j
        if (j ∈ N_r) then
            D_r = D_r ∪ j        /*node j is node destination*/
        end if
    end if
    τ_ij =(1-φ).τ_0 +φ.τ_0       /*update pheromone*/
end repeat
Prune Tree T                     /* eliminate not used link*/
```

Figure 3. Procedure to Build Tree (Algorithm 3)

4. Multi-objective Multicast Algorithm

The Multi-objective Multicast Algorithm (*MMA*), proposed in [1], is based on the *Strength Pareto Evolutionary Algorithm* (*SPEA*) [12]. This algorithm maintains an evolutionary population P and an external set of Pareto solutions P_{nd}. Starting with a random population, the individuals evolve to the desired solutions, as shown in Figure 4 [1].

```
Read multicast group (s,N_r) and traffic demand ϕ
Build routing tables
Initialize P and P_nd
while until stop criterion is not verified
    Discard identical individuals
    Evaluate individuals of P
    Update non-dominated set P_nd
    Compute fitness
    Selection
    Apply crossover and mutation
end while
```

Figure 4. General Procedure of *MMA* (*Algorithm 4*)

Build routing tables is a procedure that builds possible paths from a source s to each destination of a multicast group. It usually selects the R shortest, and R cheapest paths, where R is a parameter of the algorithm. A chromosome is represented by a string of length $|Nr|$ in which an element (gene) g_i represents a path [1], as shown in Figure 5.

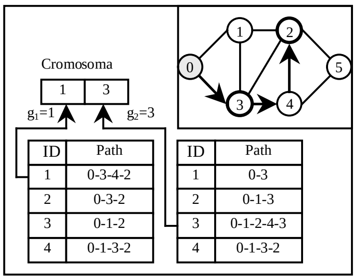

Figure 5. Relationship among a chromosome, genes and routing tables.

Initialize P and P_{nd}. generates $|P|$ chromosomes, where P is an evolutionary population. The best non-dominated solutions found so far is saved in an external set P_{nd}. Procedure *Discard identical individuals of P* replaces duplicated solutions with new randomly generated solutions, while procedure *Evaluate individuals of P* calculates the 3 objectives for each individual.

Update non-dominated set P_{nd}. include in P_{nd} non-dominated solutions of P, and it erases any dominated solution of P_{nd} . Then, fitness is computed as in [12]. The selection operator is later applied over the set $P \cup P_{nd}$, to generate a new population P. Finally, *crossover* and *mutation* operators are applied using 2-point crossover and changing some genes in each chromosome of the new population.

5. Experimental Results

Experimental tests were carried out using the NTT network [10] consisting of 55 nodes and 144 links. Four tests were performed for the 4 groups presented in Table 1. Each test consists of 3 runs for 40, 160 and 320 seconds. Both algorithms, *MOACS* and *MMA*, have been implemented on a 350 MHz AMD-K6 computer with 128 MB of RAM. The compiler used was Borland C++ V 5.02.

Table 1. Multicast Group used for the tests

	s	N_r	$\|N_r\|$
Group 1	{5}	{0, 1, 8, 10, 22, 32, 38, 43, 53}	9
Group 2	{4}	{0, 1, 3, 5, 9, 10, 12, 23, 25, 34, 37, 41, 46, 52}	14
Group 3	{4}	{0, 1, 3, 5, 6, 9, 10, 12, 17, 22, 23, 25, 34, 37, 41, 46, 47, 52, 54}	19
Group 4	{4}	{0, 1, 3, 5, 6, 9, 10, 11, 12, 17, 19, 21, 22, 23, 25, 33, 34, 37, 41, 44, 46, 47, 52, 54}	24

5.1. Comparison Procedure

The comparison procedure used for each multicast group was the following:

a) Each algorithm was run five times to calculate an average.
b) For each algorithm, five sets of non-dominated solutions were obtained (Y_1, Y_2..Y_5) and an overpopulation Y_T was calculated as the union of the five sets.
c) Dominated solutions were deleted from Y_T, forming the Pareto set of each algorithm:
   Y_{MOACS} (Pareto Front obtained of the 5 runs using *MOACS*)
   Y_{MMA} (Pareto Front obtained of the 5 runs using *MMA*)
d) A set of solutions $\hat{Y}$ was obtained as follows: $\hat{Y} = Y_{MOACS} \vee Y_{MMA}$ (8)
e) Dominated solutions were eliminated from $\hat{Y}$, to obtain an approximation of Y_{true}, called Y_{apr}[2]. Table 2 presents the number of solutions $T \in Y_{apr}$ found for every multicast group.

Table 2. Amount of Optimal Solutions for each Multicast Group.

	Group 1	*Group 2*	*Group 3*	*Group 4*
$\| Y_{apr} \|$	9	18	24	18

5.2. Results

The odd tables of each test present the average number of solutions of each algorithm that are in Y_{apr}, denoted as [$\in Y_{apr}$]. The set of solutions that are dominated by Y_{apr} is denoted as [Y_{apr}¤]. The number of found solutions is [$|Y_{alg}|$] and the percentage of solutions present in Y_{apr} is [%($\in Y_{apr}$)]. The following steps explain how to read Table 3 considering *MMA*.

a) Row Y_{MMA}, column [$\in Y_{apr}$] indicates that 5.8 solutions in average belongs to Y_{apr}.
b) Row Y_{MMA}, column [Y_{apr}¤] indicates that 0 solutions are dominates by Y_{apr}.

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[2] Note that for practical issues $Y_{apr} \approx Y_{true}$, i.e. Y_{apr} is an excellent approximation of Y_{true}.

c) Row Y_{MMA}, column [$|Y_{alg}|$] indicates that in average 5.8 solutions were found by *MMA*.

d) Row Y_{MMA}, column [%($\in Y_{apr}$)] indicates that *MMA* finds 64% of Y_{apr} solutions.

The even tables of each experiment present the covering figure among algorithms [11]. Only results for group 1 and group 4 are presented.

Experiment 1. Results for multicast group 1 (see Table 1)

a) In Tables 3, 5 and 7 *MOACS* finds almost all solutions of Y_{apr}, overcoming *MMA*.

b) All found solutions belong to Y_{apr}; therefore, the coverings are 0 in Tables 4, 6 and 8.

Table 3. Comparison with respect to Y_{apr}

	Run time 40 s.			
	$\in Y_{apr}$	Y_{apr}¤	$\lvert Y_{alg}\rvert$	%($\in Y_{apr}$)
Y_{MOACS}	8.8	0	8.8	98%
Y_{MMA}	5.8	0	5.8	64%

Table 4. Covering among algorithms

	Y_j	
Y_i	Y_{MOACS}	Y_{MMA}
Y_{MOACS}		0
Y_{MMA}	0	

Table 5. Comparison with respect to Y_{apr}.

	Run time 160 s.			
	$\in Y_{apr}$	Y_{apr}¤	$\lvert Y_{alg}\rvert$	%($\in Y_{apr}$)
Y_{MOACS}	9	0	9	100%
Y_{MMA}	5.2	0	5.2	57%

Table 6. Covering among algorithms

	Y_j	
Y_i	Y_{MOACS}	Y_{MMA}
Y_{MOACS}		0
Y_{MMA}	0	

Table 7. Comparison with respect to Y_{apr}

	Run time 320 s.			
	$\in Y_{apr}$	Y_{apr}¤	$\lvert Y_{alg}\rvert$	%($\in Y_{apr}$)
Y_{MOACS}	9	0	9	100%
Y_{MMA}	5.8	0	5.8	64%

Table 8. Covering among algorithms

	Y_j	
Y_i	Y_{MOACS}	Y_{MMA}
Y_{MOACS}		0
Y_{MMA}	0	

Experiment 4. Results for multicast group 4 (see Table 1)

a) In this last experiment characterized for a larger number of destinations multicast group, the *MOACS* also demonstrated to be better than the *MMA*. In fact, *MOACS* obtained a larger number of solutions belonging to Y_{apr}, for all run times.

b) Notice that *MOACS* solutions dominate more solutions than the *MMA* on average for 160 and 320 seconds (Tables 12 and 14); although not at 40 seconds.

Table 9. Comparison with respect to Y_{apr}

	Run time 40 s.			
	$\in Y_{apr}$	Y_{apr}¤	$\lvert Y_{alg}\rvert$	%($\in Y_{apr}$)
Y_{MOACS}	4	7.6	11.6	22%
Y_{MMA}	2.6	0.6	3.2	14%

Table 10. Covering among algorithms

	$\mathbf{Y_j}$	
Y_i	Y_{MOACS}	Y_{MMA}
Y_{MOACS}		0.2
Y_{MMA}	1.6	

Table 11. Comparison with respect to Y_{apr}

	Run time 160 s.			
	$\in Y_{apr}$	Y_{apr}¤	$\lvert Y_{alg}\rvert$	%($\in Y_{apr}$)
Y_{MOACS}	12.2	0.6	14.8	67%
Y_{MMA}	4.2	0.6	4.8	23%

Table 12. Covering among algorithms

	$\mathbf{Y_j}$	
Y_i	Y_{MOAC}	Y_{MMA}
Y_{MOAC}		0.4
Y_{MMA}	0.2	

Table 13. Comparison with respect to Y_{apr}

	Run time 320 s.			
	$\in Y_{apr}$	Y_{apr}¤	$\|Y_{alg}\|$	%($\in Y_{apr}$)
Y_{MOACS}	14	2.6	16.6	77%
Y_{MMA}	4.4	1.2	5.6	24%

Table 14. Covering among algorithms

	Y_j	
Y_i	Y_{MOACS}	Y_{MMA}
Y_{MOAC}		0.8
Y_{MMA}	0.2	

General averages of the experiments.

Tables 15 and 16 show that on average, the *MOACS* is clearly superior to *MMA*.

Table 15. Comparison with respect to Y_{apr}

	$\in Y_{apr}$	Y_{apr}¤	$\|Y_{alg}\|$	%($\in Y_{apr}$)
Y_{MOACS}	14.1	3.5	17.8	69.9%
Y_{MMA}	8.6	1.6	9.9	42.1%

Table 16. Covering among algorithms

Y_i	Y_{MOACS}	Y_{MMA}
Y_{MOAC}		0.4
Y_{MMA}	0.2	

6. Conclusions

Ant algorithms proved to be a promising approach to solve the multicast routing problem. Considering the presented experimental results, *MOACS* is able to find 69,9% of the best solutions in average, while *MMA* could only find 42.1 %. Besides, the Y_{MOACS} has a better coverage then Y_{MMA} proving its capacity to treat this kind of problems.

As future work, we will consider other objective functions, as maximum link uses and experiments with a dynamic environment and other ACO's versions.

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