

MEASUREMENT AND ANALYSIS OF POVERTY AND WELFARE USING FUZZY SETS¹

Benjamín Barán,
Universidad Nacional de Asunción

Alberto Rojas, Daniel Brites
Universidad Nacional del Este

& Leticia Barán

P.O. Box 1439
San Lorenzo – Paraguay
Fax/Tel. (595-21) 585.619
E.mail: bbaran@cnc.una.py

ABSTRACT

This paper studies the welfare and poverty in Paraguayan conurbations. It comprises the analysis of a) relative and absolute monetary poverty and welfare, b) poverty and welfare measured through physical indicators, and c) the definition and measurement of welfare and poverty as fuzzy sets. Typical poverty indices are used, making an adaptation to proper define and measure *welfare*, as a dual concept of *poverty*. However, these classical definitions have several difficulties that are overcome by introducing fuzzy definitions of *poverty* and *welfare* in an n -dimensional vector space. The new definition of welfare and poverty as fuzzy sets conveys a new approach in the effort to better represent linguistic variables that are intrinsically fuzzy. Statistical data presented in this work were taken from a Paraguayan Household Budget Survey made in 1996.

Keywords: Fuzzy Sets, Welfare, Poverty, Linguistic Variable, Poverty Line, Relative Welfare, Poverty Indices.

1. INTRODUCTION

1.1 Poverty. Historical Review

Historically, the phenomenon of poverty has been analyzed from different points of view, looking for a valid response on the factors and circumstances that give rise to it in society. Murmis M. and Feldman S. [1] assert that the study of poverty and the concern on its consequence, date back to the very beginnings of the sociological analysis. Thus, it was already the subject of sociological surveys at the end of the XVIII century, largely motivated by the belief that in industrial societies, poverty was a terrible but preventable problem. On the other hand, poverty appears in the leftist literature, both as matter of empirical analysis, such as Engels's classical study, as well as in the attempts to found the theory that capitalism would intrinsically brought misery to the working class.

In the last decades, poverty has turned back to be a central concern to analyze different social circumstances in a variety of theoretical-practical orientations [2].

1.2 Classical Definitions of Poverty

According to its primary meaning, poverty implies deprivation of something that is essential or desired. The concept of

poverty varies according to the recognized values. In one extreme, it is found the most absolute forms of poverty, as starvation or death from lack of shelter. On the other side, poverty extends continuously towards a fuzzy limit. It also varies with the wealth of societies as well as with the pass of time. Poverty appears as a multidimensional phenomenon, closely associated with the concept of exclusion. The *poverty state* is then, rather a continuum than a classical set or point on a scale of absolute values. It is defined with respect to a variety of quantitative and qualitative criteria that may change with societies and cultures. Poverty notion involves, above all, a comparative concept that refers to a relative quality. That is why there is no consensus on an absolute definition for *poverty*, even though attempts were made [3].

When talking about poverty, it is important to remark that together with material deprivations, there are other kind of deprivations in variable combinations, from one society to another. At present, it is admitted that poor people are underprivileged in several other important fields as: educational, occupational and political ones, among others [3].

1.3 Welfare and Poverty

By definition, welfare involves health, happiness, prosperity and wellbeing in general. Being so, one can conclude that it is a concept radically opposed to that of poverty and perhaps even more general, being its nature as fuzzy as poverty itself. Probably, it is because its fuzzy nature that welfare has not been studied as much as poverty.

This paper proposes the analysis of welfare as a concept dual to that of poverty, taking into account that it might be useful to know what is necessary for people to enjoy wellbeing rather to know just their degree of miseries. Following these considerations, this work presents the analysis of welfare and poverty applied to the Paraguayan conurbation, using data from a Paraguayan Household Budget Survey (PHBS) of more than 3000 family sample, made in 1996. Along the study, the household is taken as the unit of information; each household furnishes a datum on each question. The work comprises: the study of relative and absolute monetary poverty and welfare in section 2, poverty and welfare measured through physical indicators in section 3, and novel definitions and measurements of welfare and poverty as fuzzy sets, in section 4. Conclusions are left for section 5.

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2 WELFARE AND POVERTY MEASURED WITH MONETARY INDICATORS

Poverty and welfare are phenomena hard to define and still more difficult to measure, that is why there exists different tendencies and theories that try to give solutions to these subjects.

Throughout history it is found some authors, like William Graham Sumner (1883), who stated that there was no possible definition of a poor man and he deplored the use of the phrase for he deemed it too elastic and covering a pile of social fallacies. On the contrary, many scientists studied the possibility of giving an adequate definition of poverty that might serve as base for studies and later analyses. So, Robert Hunter, a contemporary to Sumner, established that the poverty can be defined and be measured in the following way: "*poor man is all person who for some reason, is incapable to provide, to itself and the people who depend on him, a standard of decent life*". After this definition, Hunter took care to determine the variables of a standard of decent life and also to calculate the income necessary to obtain it. According to his definition, poor men are all members of society whose income were below the one established as minimum [4].

Some modern economists, who also tried to define poverty, have adopted Hunter's definition, relating the income to the necessities and defining the level of necessity in different ways. For example, Galbraith (1955) considered as *extremely poor* the people whose income (although sufficient to survive) is very below those from the rest of the community. In addition, he engaged himself to define what would be the "*poverty line*" to be considered in the analysis. On the other hand, Kristol (1960) made his analysis trying to determine what would be the basic necessities of the population but at this point he met a dilemma by the fact that for each person "the basic necessities" are different, which reminds the fact that welfare and poverty involves a dose of subjectivity too [4].

The magnitude of poverty and the characteristics of the poor people are intimately related to the definition made of the poverty line: it is defined as the threshold below which people are taken as poor and above which they are taken as not poor (classical set definition). Following the same reasoning, an extension is made to this definition in order to apply it to the measurement of welfare, as a concept dual to poverty.

Referring to poverty and welfare lines, the issue of their definition needs to be considered; i.e. the level of each individual should be measured with regard to several selected resources, each of them, representative of an aspect of poverty or welfare. However, due to the fact that some of this aspects are too hard to measure, if not impossible (deprivation of some item could obey to subjective reasons), monetary indicators as expenditure, wealth and income are usually preferred [5].

There are two ways of approaching the monetary reference measurement. One way is just trying to determine how well is a household according to a monetary indicator enjoyed, the *absolute poverty*; so it is necessary to set the poverty and welfare line to some reasonable level. The price of a basket of essential needs can be considered for that purpose, as introduced by Rowntree (1901). However, because difficulties in selecting the contents of such basket, at the present work the Orshansky line is used; that is, the price of a basket of nutrients, based on the assumption that the minimum total

needs of an individual is proportional to his basic needs of food. On the other hand, there is the *relative poverty*, the measure of deprivation relative to the standard of society (or its mean value); what it really measures is rather inequality, i.e., the distribution of monetary poverty [5].

Again, extensions of absolute poverty and relative poverty are made, introducing the concepts of *absolute welfare* and *relative welfare* as dual of the first two.

In general, studies are made through several indices that measure different characteristics of the phenomena; even though, none of them is a perfect indicator of the studied phenomena. For that porpoise, alimentary expenditure and total income are chosen as independent variables for measuring several indices. In this section, they are not perfect indicators: rich people tend to underreport their income while households with similar levels of permanent income can be very different in their expenditure pattern. Nevertheless, both are calculated in the preset work [5].

Once the household is chosen as the unit of analysis, it appears the problem of *equivalence scales*: a family of three members with the same income than a family of eight is likely to be far better off. The expenditure and income per capita are calculated dividing expenditure and income, respectively, by the amount of members of the household. The data analysis presented here is based on this scale. However, it is important to remark the *economy of scale* that appears when people living together share their resources: more crowded households need less than proportional extra income per member in order to enjoy the same standard of living of small households [5]. Economy of scale can be evaluated using the so called OECD scale, very often used in EUROSTAT studies which gives a weight of 1 to the first member and 0.7 to any other member if he/she is over thirteen, otherwise (children) members are weighted 0.5. This scale makes small families look poorer. Some indices are calculated in this study using the OECD scale in order to compare figures.

2.1 Poverty Indices

The average monthly alimentary expenditure per capita of Paraguayan conurbation households according to PHBS'96 is 137,005.85 guaranías. At the same time, the average monthly total income calculated is 566,613.34 guaranías; i.e. more than 4 times the average alimentary expenditure.

Table 1 shows most recognized indices used in poverty studies, for three different poverty lines (thresholds):

- 50% of the average (poor);
- 40% of the average (very poor); and
- 25% of the average (extremely poor).

The variables used for the calculations are per capita

- a) alimentary expenditure, and
- b) total income.

The first column of Table 1 shows the *headcount ratio*, H , i. e., the proportion of poor households according to the corresponding threshold applied:

$$H = \frac{q}{n}, \quad (1)$$

where:

- q : amount of poor households,
- n : amount of households.

If one is interested in knowing how far are these poor households from the threshold (poverty intensity), the *average income ratio*, I, in the second column gives this information although it says nothing about the proportion of poor households:

$$I = \frac{T - \bar{X}}{T}, \quad (2)$$

where:

T : threshold value,
 \bar{X} : average.

These two indices give only a partial view on the phenomenon. Some other indices with better properties and covering the aspects measured by H and I have already been introduced. Two of them are presented here: the Hagenaars index, HAG, and the Foster, Greer and Thorbecke indices with poverty aversion parameter α equal to 2, 3 and 4, i. e., FGT₂, FGT₃ and FGT₄ respectively; where:

$$HAG = \frac{q \log T - \log \mu}{n \log T}, \quad (3)$$

with:

μ : geometric average of poor households, and

$$FGT_\alpha = \frac{1}{n} \sum \left(\frac{T - x_i}{T} \right)^{\alpha-1} \quad (4)$$

with

x_i : expenditure/income of household i , $i=1, 2, \dots, n$.

These indices are placed orderly in columns following those of H and I of Table 1. It is easy to prove that:

$$FGT_2 = H \cdot I. \quad (5)$$

TABLE 1: Monetary indices of relative poverty.

a) Alimentary expenditure per capita

Threshold: 50% of average					
H	I	HAG	FGT ₂	FGT ₃	FGT ₄
0.164	0.228	0.004	0.037	0.014	0.007
Threshold: 40% of average					
0.080	0.209	0.002	0.017	0.006	0.003
Threshold: 25% of average					
0.013	0.283	0.000	0.004	0.014	0.001

b) Total income per Capita

Threshold: 50% of average					
H	I	HAG	FGT ₂	FGT ₃	FGT ₄
0.394	0.402	0.020	0.158	0.087	0.081
Threshold: 40% of average					
0.302	0.369	0.014	0.111	0.059	0.037
Threshold: 25% of average					
0.136	0.356	0.006	0.048	0.026	0.016

H: head count ratio
 I: income gap ratio
 HAG: Hagenaars
 FGT: Foster, Greer and Thorbecke

Table 2 displays indices H, I and FGT₂ for per capita and OECD scales. Note how index H is markedly better in OECD scale. This can be explained by observing that Paraguayan poor households are considerably more crowded than average and taking into account that OECD scale favors crowded households; as a matter of fact: being 4.41 the average of household members in study, it rises to 6.11 among poor households at the threshold of 50% of expenditure average. Index I remains little affected by this change of scale. Observe how the ratio of poor households as measured on alimentary expenditure terms is pretty better than that got on total income terms, the reason for that could be that poorest households tend to allot relatively larger proportions of their budget to food expenses, escaping this way from appearing below poverty thresholds. Finally, index FGT₂ again reflects the phenomenon observed with index H, as expected.

TABLE 2 RELATIVE POVERTY: robustness of the proportion of poor to changes with different scales.

a) Alimentary expenditure

Threshold: 50% of average		
	Per capita	OECD
H	0.164	0.122
I	0.228	0.217
FGT ₂	0.037	0.026
Threshold: 40% of average		
	Per capita	OECD
H	0.080	0.053
I	0.209	0.226
FGT ₂	0.017	0.012
Threshold: 25% of average		
	Per capita	OECD
H	0.013	0.010
I	0.283	0.349
FGT ₂	0.004	0.003

b) Total income

Threshold: 50% of average		
	Per capita	OECD
H	0.394	0.367
I	0.402	0.376
FGT ₂	0.158	0.138
Threshold: 40% of average		
	Per capita	OECD
H	0.302	0.265
I	0.369	0.356
FGT ₂	0.111	0.094
Threshold: 25% of average		
	Per capita	OECD
H	0.136	0.111
I	0.356	0.362
FGT ₂	0.048	0.040

As stated before, this study takes the price of an established essential basket of nutrients as the reference for measuring absolute poverty and welfare; such basket price was established as 122,692 guaranias in 1996. Table 3 shows indices H, I and FGT₂ measured for absolute poverty.

TABLE 3: Absolute poverty index measurements based on the alimentary basket*

	Expenditure per capita	Total income per capita
H	0.025	0.017
I	0.296	0.443
FGT ₂	0.007	0.008

* Threshold: price of an established essential basket of nutrients

2.2 Welfare Indices

Because in general, welfare indices are not studied in literature, this section proposes an extension of the already known indices for poverty, making possible the analysis of welfare as a mathematically dual phenomenon. To define the similar indexes for welfare, some adaptation should be done to Eq. (2) to (4) to proper measure welfare:

$$I = \frac{\bar{X} - T}{T}, \quad (6)$$

$$HAG = \frac{q}{n} \frac{\log \mu - \log T}{\log T}, \quad (7)$$

$$FGT_{\alpha} = \frac{1}{n} \sum \left(\frac{x_i - T}{T} \right)^{\alpha-1} \quad (8)$$

As a consistency proof, it is easy to prove that Eq. (5) holds.

To obtain a dual for the threshold values, it is enough to consider the poverty lines used in Table 1, but above the average instead. Table 4 shows the above indices for the following three (dual) welfare lines:

- 150% of the average (welfare);
- 160% of the average (good welfare); and
- 175% of the average (extremely good welfare).

Index H reveals that moving threshold from 150% to 175%, gives a markedly smaller variation in the number of welfare households (a ratio of just 1.49 for alimentary expenditure figures) as compared with the variation found for poor households when moving from 50% to 25% thresholds (their dual thresholds, with ratio of 12.6), showing an asymmetrical characteristic. Besides, in terms of alimentary expenditure, poverty households start off with 0.164 of the population, more than the 0.148 for welfare households; this behavior suggests a distribution pattern of few households that are very rich and many poor households. When comparing total incomes, the same conclusions may be drawn.

TABLE 4: Monetary indices of relative welfare.

a) Expenditure per capita					
Threshold: 150% of average					
H	I	HAG	FGT ₂	FGT ₃	FGT ₄
0.148	0.501	0.004	0.074	0.091	0.178
Threshold: 160% of average					
0.127	0.478	0.003	0.061	0.072	0.133
Threshold: 175% of average					

0.099 0.463 0.003 0.046 0.051 0.087

b) Income per capita

Threshold: 150% of average					
H	I	HAG	FGT ₂	FGT ₃	FGT ₄
0.163	1.053	0.006	0.172	0.885	11.277
Threshold: 160% of average					
0.150	1.006	0.006	0.151	0.758	9.148
Threshold: 175% of average					
0.129	0.982	0.005	0.126	0.611	6.832

Table 5 compares per capita scale with OECD scale. Here the phenomenon of bettering shift of scale is absolutely negligible; this is perhaps, because rich families tend to be less crowded than average (2.56 for a 150% threshold, compared to the average of 4.41).

Conspicuously, as α grows, FGT indexes assume notably large values; this obeys to the remarked fact that rich households are notably beyond average figures, making the term $((x_i - T)/T)^{\alpha-1}$ in Eq. (8) grow as α does.

TABLE 5 RELATIVE WELFARE: robustness of the proportion of changes with different scales.

a) Expenditure

Threshold: 150% of average		
	Per capita	OECD
H	0.148	0.140
I	0.501	0.391
FGT ₂	0.074	0.055
Threshold: 160% of average		
	Per capita	OECD
H	0.127	0.115
I	0.478	0.379
FGT ₂	0.061	0.044
Threshold: 175% of average		
	Per capita	OECD
H	0.099	0.083
I	0.463	0.375
FGT ₂	0.046	0.031

b) Income

Threshold: 150% of average		
	Per capita	OECD
H	0.163	0.164
I	1.053	0.967
FGT ₂	0.172	0.159
Threshold: 160% of average		
	Per capita	OECD
H	0.150	0.145
I	1.006	0.961
FGT ₂	0.151	0.139
Threshold: 175% of average		
	Per capita	OECD
H	0.129	0.124
I	0.982	0.931
FGT ₂	0.127	0.115

Table 6 shows indices H, I and FGT₂ measuring for absolute welfare, simply defined as K times the absolute poverty line (see Table 3).

TABLE 6: Absolute welfare index measurements based on the alimentary basket*

	Expenditure per capita	Total income per capita
H	0.001	0.165
I	0.266	0.007
FGT ₂	0.000	0.001

* **Threshold:** 20*price of an established essential basket of nutrients.

3 MEASUREMENT THROUGH PHYSICAL INDICATORS

In order to supply some objective information on the deprivation level endured by poor households and possession level enjoyed by welfare households, Tables 7, 8 and 9 present the results found for possession of 12 home appliances chosen from the PHBS'96. At this point, it should be mentioned that it is desirable to have a more comprehensive list of items, in order to better indicate poverty and welfare. For example, items that could be added include: computer, Internet access, cellular telephone, cable television, freezer, drier, dishwasher, etc. (but these items were not included in the PHBS'96). Table 7 provides information for all households, while Tables 8 and 9 provides the same information but for thresholds of poverty and welfare respectively.

TABLE 7: ratio of households that own housing equipment.

Appliance	Households (% of total)
Refrigerator	76.2%
Stove	88.1%
Microwave oven	5.3%
Washing machine	44.7%
Sewing machine	27.2%
Television set	85.3%
Video recorder	20.1%
Hi-fi Equipment	41.0%
Air conditioner	15.0%
Bicycle	43.6%
Motorcycle	7.3%
Car/pick-up	26.2%

This information falls on the absolute poverty and welfare domain since most of the items considered serve as economic indicators regardless of the particular society studied. However, different studies may choose different lists of items.

TABLE 8 ratio of poor households that own household equipment.

Appliance	Households		
	(a)	(b)	(c)
Refrigerator	59.6%	54.5%	38.7%
Stove	71.2%	64.6%	48.4%
Microwave oven	0.8%	0.5%	3.2
Washing machine	31.1%	28.6%	16.1
Sewing machine	23.1	20.6	16.1

Television set	72.0	66.6	41.9
Video recorder	7.5	6.3	9.7
Hi-fi Equipment	23.8	20.6	19.4
Air conditioner	4.1	3.2	6.5
Bicycle	44.3	38.1	29.0
Motorcycle	5.2	2.1	0.0
Car/pick-up	12.2	12.2	6.5

(a): Poor for threshold of 50% of expenditure average
 (b): Poor for threshold of 40% of expenditure average
 (c): Poor for threshold of 25% of expenditure average

TABLE 9: Ratio of welfare households that own household equipment.

Appliance	Households		
	(a)	(b)	(c)
Refrigerator	78.5	78.0	77.8
Stove	89.4	89.4	88.0
Microwave oven	11.5	11.3	10.7
Washing machine	46.4	47.7	44.4
Sewing machine	23.2	23.7	20.9
Television set	84.5	84.0	82.9
Video recorder	27.5	28.3	27.4
Hi-fi Equipment	48.7	48.7	47.0
Air conditioner	28.7	29.3	28.6
Bicycle	31.8	31.3	27.8
Motorcycle	7.2	7.3	7.3
Car/pick-up	35.5	36.3	35.5

(a): For threshold of 150% of expenditure average
 (b): For threshold of 160% of expenditure average
 (c): For threshold of 175% of expenditure average

4 PROPOSAL USING FUZZY SETS

Given the difficulties in having a good definition of *poverty* (and its dual: *welfare*) using classical set, because of its ambiguity, this section introduces a new definition based on fuzzy set theory. This way, welfare and poverty may benefit from all the research done with linguistics variables [6,7].

4.1 Review of Fuzzy Sets

The encapsulation of objects into a collection whose members share some general features or properties naturally implies the notion of a set. Several sets or categories used in describing real-world objects do not possess well-defined boundaries. Consider, for example, notions or concepts such as *high* salary, *populous* city, *accurate* clock, *high* temperature and so forth in which the italicized words identify the sources of fuzziness. Whether an object belongs to such a category is a matter of degree, expressed, for example, by a real number in the unit interval [0, 1]. The closer that number is to 1, the higher the grade of the object membership in the particular set. Zadeh L.(1965) formally defined a fuzzy set as follows [6]:

A fuzzy set is characterized by a *membership function* mapping the elements of a domain, space, or universe of discourse X to the unit interval [0, 1]. That is, $A:X \rightarrow [0, 1]$.

Thus, a fuzzy set A in X , may be represented as a set of ordered pairs of a generic element $x \in X$ and its grade of membership: $A = \{(A(x)/x) | x \in X\}$. Clearly, a fuzzy set is a generalization of the concept of a set whose membership function takes only two values: $\{0, 1\}$. Contrary to the qualitative symbolic role of numbers 1 and 0 in characteristic functions of classical sets, numbers involved in membership functions of fuzzy sets have a quantitative meaning, by representing the degree of membership of an element x to a given set.

In principle, any function of the form $A: X \rightarrow [0, 1]$ describes a membership function associated with a fuzzy set A . A useful representation depends on the concept to be represented and on its context. In certain cases, however, the meaning semantics captured by fuzzy sets is not too sensitive to variations in the shape, and simple functions are convenient. In many practical instances fuzzy sets can be represented explicitly by families of parametric functions (Ex.: triangular, Gaussian, exponential, etc.). An important class of membership functions is trapezoidal shaped, which is captured by the generic graphical representation in Fig. 1.

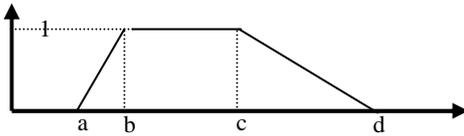


Figure 1: Trapezoidal shaped membership function

Each function in this class is fully characterized by the four parameters, a, b, c and d , via the generic form:

$$A(x) = \begin{cases} \frac{a-x}{a-b} & \text{when } a \leq x \leq b \\ 1 & \text{when } b \leq x \leq c \\ \frac{d-x}{d-c} & \text{when } c \leq x \leq d \\ 0 & \text{otherwise} \end{cases}$$

For each of the three basic operations on classical sets: complement, union and intersection; there exists a broad class of operations that qualify as their fuzzy generalizations. However, one special operation in each of the three classes possesses certain desirable properties, which often make it a good approximation of the respective linguistic term. These special operations on fuzzy sets, which are referred to as *standard fuzzy operations*, are by far the most common operations in practical applications of fuzzy set theory and they are: standard fuzzy complement, defined by the formula

$$\bar{A}(x) = 1 - A(x), \quad (9)$$

standard fuzzy union, and standard fuzzy intersection.

There are several ways of representing fuzzy sets: graphs, tables, lists, mathematical formulae, or classical coordinates in a n -dimensional unit cube. There is also a representation based on specific assignments of numbers in $[0, 1]$ to crisp (classical) sets. A given fuzzy set X is always associated with a family of crisp subsets of X . Each of these subsets consists of all elements of X whose membership degrees in the fuzzy set are restricted to some given crisp subset of $[0, 1]$. One way

of restricting membership degrees is particularly important. It is a restriction of membership degrees that are greater than or equal to some chosen value α in $[0, 1]$. When this restriction is applied to a fuzzy set A one obtains a crisp subset ${}^\alpha A$ of the universal set X , which is called an α -cut of A . Formally,

$${}^\alpha A(x) = \{x \in X | A(x) \geq \alpha\}. \quad (10)$$

Any fuzzy set may be completely characterized by its α -cuts [7]:

$$A = \bigcup_{\alpha \in [0,1]} {}^\alpha A(x) \quad (11)$$

where

$${}^\alpha A(x) = \alpha \quad {}^\alpha A(x), \text{ and}$$

\bigcup denotes the standard fuzzy union.

Another important concept within fuzzy set theory is that of linguistic variables. A linguistic variable is a variable whose values are words or sentences rather than numbers. The essential motivations for using linguistic variables are [6]:

- they may be regarded as a form of information compression called granulation (Zadeh 1994),
- they serve as a means of approximate characterization of phenomena that are either too ill-defined or too complex, or both, to permit a description in sharp terms (Zadeh 1975) and
- they provide a means for translating linguistic descriptions into numerical, computable ones. Therefore, the duality between symbolic and numerical processing becomes natural instead of antagonistic.

In each application of fuzzy set theory, it must be constructed appropriate membership functions by which the intended meanings of relevant linguistic terms are adequately captured. This is a problem of knowledge acquisition that involves one or more experts in the application area and a knowledge engineer to extract the knowledge of interest from the experts and to express it in some operational form [7].

Finally, the *Extension Principle* is introduced to transform fuzzy sets via functions [6]. Let \mathbf{X} and \mathbf{Y} be two sets and f a mapping from \mathbf{X} to \mathbf{Y} :

$$f: \mathbf{X} \text{ to } \mathbf{Y}.$$

Let A be a fuzzy set in \mathbf{X} . The extension principle states that the image of A under this mapping is a fuzzy set $B = f(A)$ in \mathbf{Y} such that, for each $y \in \mathbf{Y}$,

$$B(y) = \sup_{x \in \mathbf{X}} A(x), \quad (12)$$

subject to $x \in \mathbf{X}$ and $y=f(x)$.

Note that \mathbf{X} may be a vector with several fuzzy components; thus, a fuzzy set as poverty $P(\mathbf{X})$ may be defined by the *Extension Principle* as a function of the n components of \mathbf{X} .

4.2 Poverty and Welfare as Fuzzy Sets

Classical studies of poverty may be criticized because a person who earns 1 cent above the threshold is not considered poor while another who earns 1 cent less than the threshold is defined as poor; however, *there is no appreciable difference in their quality of living*. This issue takes root in the very nature of poverty and welfare as fuzzy concepts; therefore, this section approaches these concepts according to their very nature, taking advantage of the ability of fuzzy theory to deal with vagueness and language flexibility.

In this context, poverty is defined as a fuzzy set; thus, a person represented by his characteristic vector \mathbf{X} , has a membership function $P: \mathbf{X} \rightarrow [0,1]$, that gives his level of membership to the set of poor people, smoothing the huge difference of classical definitions between the two persons with an income differing in just 2 cents, mentioned above. In fact, using the poverty definition presented in this section, the two persons would have almost the same membership function, instead of being considered one as poor and the other not.

Analogously, welfare may be defined by Eq. (9) as a complementary fuzzy set of poverty; that is, $W: \mathbf{X} \rightarrow [0,1]$, with:

$$W(\mathbf{X}) = 1 - P(\mathbf{X}). \quad (13)$$

The starting point for calculating $P(\mathbf{X})$ and its complement $W(\mathbf{X})$ is a suitable definition of the characteristic vector \mathbf{X} , that we define as a n -dimensional fuzzy vector; that is,

$$\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n]^T \quad (14)$$

where \mathbf{x}_k is a fuzzy variable that may represent any economical, cultural, social or environmental factor. As an example, \mathbf{x}_k may be: family income, income per (weighted) capita, expenditure, educational level, housing, household equipment and appliances, medical care, retirement pension, occupation, car ownership or services as running water, electricity, telephone and cable TV, just to mention a few.

For each fuzzy variable \mathbf{x}_k a (half-trapezoidal) membership function P_k (or W_k) is defined as:

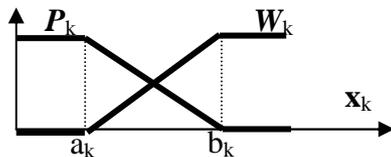


Figure 2. Poverty and Welfare as complementary fuzzy sets.

$$P_k(\mathbf{x}_k) = \begin{cases} 1 & \dots \dots \dots \mathbf{x}_k \leq a_k \\ (\mathbf{x}_k - b_k) / (a_k - b_k) & \dots \dots a_k \leq \mathbf{x}_k \leq b_k \\ 0 & \dots \dots \dots b_k \leq \mathbf{x}_k \end{cases}$$

where a_k represents the poverty threshold and b_k the welfare threshold for the given variable \mathbf{x}_k . As an example, if \mathbf{x}_k is the family income, a_k may be the basic basket value and b_k may be an appropriate multiple of this value. Table 10 presents the 19 fuzzy variables taken into account in the present study, using PHBS'96 data, with their respective poverty and welfare thresholds, defined above.

To calculate the membership function $P(\mathbf{X})$ for the given characteristic vector \mathbf{X} , the *Extension Principle* may be used. For that purpose, a linear weighted sum of the $n = 19$ fuzzy components of \mathbf{X} are defined as:

$$P(\mathbf{X}) = \sum \omega_k P_k \quad \text{with } \sum \omega_k = 1, \quad (15)$$

where the ω_k are relative weights defined by experts, considering the relative importance of each factor taken into account. That way, for a given household with characteristic vector \mathbf{X} , poverty and welfare membership values may be calculated. Third column of Table 10 gives the relative weights used for the present study, calculated after consulting

several sociologists who ranked each variable in a scale from 0 (no matter) to 10 (extremely important).

After calculating a membership function value for each household surveyed in the PHBS'96, the following figures were found:

- $\bar{w} = 0.5162$ for welfare, with variance = 0.0246, and
- $\bar{p} = 1 - \bar{w} = 0.4838$ for poverty.

Figure 3 presents the distribution of the number of households for different membership function intervals, resembling a Gaussian function, what may be explained by the *Central Limit Theorem* [8] applied to Eq. (15).

Figure 4 shows the amount of households members of different α -cuts, showing that for any value of α , the number of households with welfare outcomes the one for poor households. This unexpected result for an underdeveloped country may be explained by the fact that the PHBS'96, by design excludes population not residing in houses. As a result, the homeless and people living in collective dwellings are excluded from the analyzed data.

There is an interesting graphical interpretation for the average values of welfare and poverty given above. In fact, it can be proved that they represent the ratio of the areas under curves of Figure 4, to the total number of households. Formally:

$$\bar{w} = \frac{\sum (x_i \Delta \alpha)}{\sum x_i} \quad \text{and} \quad \bar{p} = \frac{\sum (y_i \Delta \alpha)}{\sum y_i}$$

where x_i and y_i are elements of the sets ${}^\alpha W$ and ${}^\alpha P$ respectively and $\sum x_i$ and $\sum y_i$ represent the total number of households. This result suggests a possible indicator (In) of the soundness of a society that could be defined as

$$\text{In} = \bar{w} - \bar{p} = \bar{w} - (1 - \bar{w}) = 2\bar{w} - 1 \quad (16)$$

ranging from 1 to -1; i. e., from absolute comfort to absolute deprivation. For Paraguayan conurbations this indicators equals 0.0324, indicating that welfare exceeds poverty (in more than 3%), probably because of the reasons mentioned above.

5. CONCLUSIONS

The paper reviewed classical definitions of poverty and the corresponding indices used in sociological studies of poverty, extending this definition and the use of indices to the dual concept of welfare. That way, the inclusion of welfare as a more adequate concept to describe what people need to live comfortably, is introduced.

Analyzing traditional indices for poverty and its extended concept to welfare using crisp sets, a lot of arguments arises. Therefore, a novel definition of welfare and poverty as complementary fuzzy sets is introduced. With the new ideas, interesting results are obtained, even though several limitations with the experimental data is recognized, as participation is voluntary, there is a margin of non response, usually people with high levels of income happen to be the most reluctant in answering or they underreport their incomes. People not being able to read and write fluently, also tend to be uncooperative. So, some poverty pockets, probably, are underrepresented. For a future work, it would be desirable to design a special questionnaire to measure every aspect of human life concerning welfare and poverty.

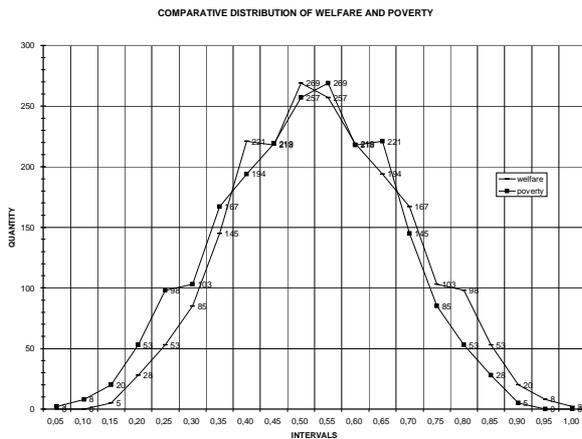


Figure 3: Fuzzy welfare and poverty distribution, calculated with data taken from the PHBS'96.

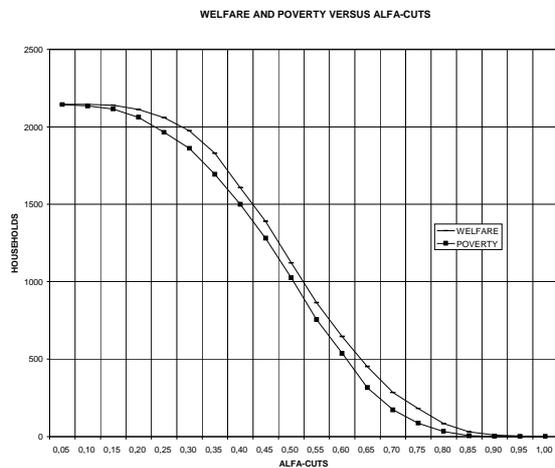


Figure 4: Amount of households that are members of a given α -cut, for different values of α .

The new approach is especially convenient for studying welfare and poverty, taking advantage of this novel and powerful branch of mathematical analysis that has the tools to deal with linguistic variables. At the same time, a door was open for measuring welfare and poverty as a general function of n independent variables that may represent any aspect of well being, better reflecting all aspects of welfare as: culture, health, wealth, etc., all without loss of individuality.

This is perhaps only a small step in a new direction of social research. New studies are going to consolidate it, enhancing its scope and utility for a better comprehension and treatment of social phenomena.

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TABLE 10: Fuzzy components of vector X .

Variable	Description	Relative Weight $[w_k]$	Poverty Threshold $[a_k]$	Welfare Threshold $[b_k]$
x_1	Housing	0.0364	Improvised dwelling	Apartment
x_2	Wall Quality	0.0083	Sun-dried brick	Stone
x_3	Floor Quality	0.0033	Wood	Ceramic
x_4	Water	0.0606	Water well	Running
x_5	Electricity	0.0364	No	Yes
x_6	Bathroom	0.0116	No	w/shower
x_7	Sanitary Services	0.0165	Latrine	WC connected
x_8	Kitchen	0.0149	Bonfire	room w/stove
x_9	Rubbish Deposition	0.0121	Yard deposition	Public gathering
x_{10}	House ownership	0.0727	Transitory occupant	Ownership
x_{11}	Appliances & Car/pick up	0.1273	Only one	10 out of 12
x_{12}	Education	0.0364	Incomplete Elementary	University
x_{13}	Health Insurance	0.0545	Public insurance	Private insurance
x_{14}	Retirement Pension	0.0727	No	Yes
x_{15}	Working activity	0.0908	Unemployed	Employer
x_{16}	Income /Capita	0.0606	A/5	20 times a_k
x_{17}	Income OECD	0.1212	0.3 A	20 times a_k
x_{18}	Expenditure /C	0.674	0.2 A	20 times a_k
x_{19}	OECD Income	0.963	0.3 A	20 times a_k

A = Alimentary basket.

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