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# Reactive Power Compensation using a Multi-objective Evolutionary Algorithm

Benjamín Barán, *Member, IEEE*, José Vallejos, Rodrigo Ramos and Ubaldo Fernández, *Member, IEEE*.

***Abstract* — This paper presents a new approach to treat Reactive Power Compensation in Power Systems using a Multi-objective Optimization Evolutionary Algorithm. A variant of the Strength Pareto Evolutionary Algorithm is proposed to independently optimize several parameters instead of traditional constrained Single-objective approach where an objective function is a linear combination of several factors, such as, investment and transmission losses, with several constrains that limit other parameters as reliability and voltage profile.**

**With the proposed approach, a set of optimal solutions known as Pareto set is found before deciding which solution best combines different features. This set is compared with a set of compensation schemes elaborated by a team of specialized engineers, using appropriate test suit metrics.**

**Comparison results emphasize outstanding advantages of the proposed computational approach, such as: ease of calculation, better defined Pareto Front and a larger number of Pareto solutions.**

***Index Terms* — Reactive Power Compensation, Multi-objective Optimization, Evolutionary Algorithms.**

## I. Introduction

REACTIVE Power Compensation is commonly addressed as a constrained *Single-objective Optimization Problem* (SOP) [1-3]. With this approach, an adequate location and size of shunt capacitor banks are found. Traditionally, the objective function is a linear combination of several factors, such as investment and transmission losses, subject to operational constrains as reliability and voltage profile [4]. *Single-objective Optimization Algorithms* usually provide a unique optimal solution. On the contrary, *Multi-objective Optimization Evolutionary Algorithms* (MOEA) independently and simultaneously optimize several parameters turning most traditional constraints into new objective functions. This seems more natural for real world problems where choosing a threshold may seem arbitrary [5]. As a result, a wide set of optimal solutions (known as Pareto set) may be found. Therefore, an engineer may have a whole set of optimal alternatives before deciding which solution is the best compromise of different (and sometimes contradictory) features.

To solve the Reactive Power Compensation Problem, this paper presents a new approach based on the *Strength Pareto Evolutionary Algorithm* (SPEA) [7]. This is a MOEA with an external population of Pareto Optimal solutions that best conforms a Pareto Front, provided by a clustering process that saves the most representative solutions [8].

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## II. Multi-objective Optimization Problems

A general *Multi-objective Optimization Problem* (MOP) [9] includes a set of $n$ decision variables, a set of $k$ objective functions, and a set of $m$ restrictions. Objective functions and restrictions are functions of decision variables. This can be expressed as:

$$\begin{array}{ll} \textit{Optimize} & \mathbf{y} = \mathbf{F}(\mathbf{x}) = [F_1(\mathbf{x}) \quad F_2(\mathbf{x}) \quad \cdots \quad F_k(\mathbf{x})] \\ \textit{s.t.} & \mathbf{e}(\mathbf{x}) = [e_1(\mathbf{x}) \quad e_2(\mathbf{x}) \quad \cdots \quad e_m(\mathbf{x})] \geq \mathbf{0} \\ \textit{where} & \mathbf{x} = [x_1 \quad x_2 \quad \cdots \quad x_n] \in X \\ & \mathbf{y} = [y_1 \quad y_2 \quad \cdots \quad y_k] \in Y \end{array} \qquad (1)$$

$\mathbf{x}$ is known as decision vector and $\mathbf{y}$ as objective vector. $X$ denotes the decision space and the objective space is denoted by $Y$. Depending on the problem at hand "optimize" could mean minimize or maximize.

The set of restrictions $\mathbf{e}(\mathbf{x}) \geq \mathbf{0}$ determines the set of feasible solutions $X_f$ and its corresponding set of feasible objective vectors $Y_f$.

From this definition, it follows that every solution consists of a n-tuple $\mathbf{x}$, that yields an objective vector $\mathbf{y}$, where every $\mathbf{x}$ must satisfy the set of restrictions $\mathbf{e}(\mathbf{x}) \geq \mathbf{0}$. The optimization problem consists in finding the $\mathbf{x}$ that has the "best" $\mathbf{F}(\mathbf{x})$. In general, there is not one "best" solution, but a set of solutions, none of which can be considered better than the others if all objectives are considered at the same time. This derives from the fact that there could be (and mostly there are) conflicts between the different objectives that compose a problem. Thus, a new concept of optimality should be established for MOPs.

In common SOPs the set of feasible decision variables is completely ordered by the objective function $F$. The goal is simply to find the value (or set of values) that lead to the optimal values of $F$. In contrast, in multi-objective optimization the feasible decision vector set is only partially ordered; i.e., there exist a decision vector $\mathbf{x_1}$ and a decision vector $\mathbf{x_2}$ and $\mathbf{F}(\mathbf{x_1})$ cannot be considered better than $\mathbf{F}(\mathbf{x_2})$, neither $\mathbf{F}(\mathbf{x_2})$ is better than $\mathbf{F}(\mathbf{x_1})$. Then, mathematically the relations =, $\leq$ and $\geq$ should be extended. This could be done using the concept of *dominance* as explained below. In fact, given two decision vectors **u, v** $\in X$ in a context of minimization, we can state:

$$\mathbf{F}(\mathbf{u}) = \mathbf{F}(\mathbf{v}) \quad \textit{iff} \quad F_i(\mathbf{u}) = F_i(\mathbf{v}) \quad \forall i \in \{1, 2, \ldots, k\}$$
$$\mathbf{F}(\mathbf{u}) \leq \mathbf{F}(\mathbf{v}) \quad \textit{iff} \quad F_i(\mathbf{u}) \leq F_i(\mathbf{v}) \quad \forall i \in \{1, 2, \ldots, k\} \quad (2)$$
$$\mathbf{F}(\mathbf{u}) < \mathbf{F}(\mathbf{v}) \quad \textit{iff} \quad \mathbf{F}(\mathbf{u}) \leq \mathbf{F}(\mathbf{v}) \wedge \mathbf{F}(\mathbf{u}) \neq \mathbf{F}(\mathbf{v});$$

where $\wedge$ denotes an *and* operation.

The relations $\geq$ and > could be defined in similar ways. Then, given two decision vectors of a MOP, $\mathbf{x_1}$ and $\mathbf{x_2}$ they comply to one of three possible conditions:

- either $\mathbf{F}(\mathbf{x}_1) < \mathbf{F}(\mathbf{x}_2)$,
- or $\mathbf{F}(\mathbf{x}_2) < \mathbf{F}(\mathbf{x}_1)$,
- or $\mathbf{F}(\mathbf{x}_1) \nleq \mathbf{F}(\mathbf{x}_2) \wedge \mathbf{F}(\mathbf{x}_2) \nleq \mathbf{F}(\mathbf{x}_1)$.

The above relations may be expressed with the following symbols:

*Pareto Dominance*. Given two objective vectors **a**, **b**$\in X$

$$\mathbf{a} \succ \mathbf{b} \ (\mathbf{a} \textit{ dominates } \mathbf{b}) \quad \textit{iff} \quad \mathbf{a} < \mathbf{b}$$
$$\mathbf{b} \succ \mathbf{a} \ (\mathbf{b} \textit{ dominates } \mathbf{a}) \quad \textit{iff} \quad \mathbf{b} < \mathbf{a} \quad (3)$$
$$\mathbf{a} \sim \mathbf{b} \ (\mathbf{a} \textit{ and } \mathbf{b} \textit{ are not comparable}) \quad \textit{iff} \quad \mathbf{a} \nleq \mathbf{b} \wedge \mathbf{b} \nleq \mathbf{a}$$

Definitions for the maximization and maximization/minimization problems could be formulated in a similar way.

At this point the concept of Pareto optimality can be introduced. A solution is said to be Pareto optimal or "non inferior" if any objective can not be improved without degrading others.

*Pareto Optimality*. A decision vector $\mathbf{x} \in X_f$ and its corresponding objective vector $\mathbf{y} = \mathbf{F}(\mathbf{x}) \in Y_f$ is non-dominated with respect to a set $A \subseteq X_f$ if and only if

$$\forall \ \boldsymbol{a} \in A : (\boldsymbol{x} \succ \boldsymbol{a} \vee \boldsymbol{x} \sim \boldsymbol{a}) \quad (4)$$

where $\vee$ denotes an *or* operation.

When **x** is non-dominated with respect to the whole set $X_f$ (and only in this case) **x** is a *Pareto optimal solution*. The whole set of Pareto optimal solutions is known as Pareto optimal set *P*; i.e.

$$P = \{ \mathbf{x} \in X_f \mid \mathbf{x} \succ \mathbf{v} \vee \mathbf{x} \sim \mathbf{v} \quad \forall \ \mathbf{v} \in X_f \} \quad (5)$$

The corresponding set of objective vectors **y** is known as *Pareto optimal front FP*; i.e.,

$$FP = \{ \mathbf{y} \in Y_f \mid \mathbf{y} = \mathbf{F}(\mathbf{x}) \quad \forall \ \mathbf{x} \in P \} \quad (6)$$

Dealing with Pareto optimal solutions, it is clear that they are non-comparable. This points to the fact that a MOP does not always have a single solution, but a set of compromise solutions. None of these solutions can be defined as "the best", unless other information is added (as a weight for every objective).

## III. Mathematical formulation

For the purposes of this paper, the following assumptions where considered in the formulation of the problem:

- shunt-capacitor bank cost per MVAr is the same for all busbars of the power system;
- power system is considered only at peak load.

Based on these considerations [4, 10], four objective functions $F_i$ (to be minimized) have been identified for the present work: $F_1$ and $F_2$ are related to investment and transmission losses, while $F_3$ and $F_4$ are related to quality of service. The objective functions to be considered are:

$F_1$: *Investment in reactive compensation devices*

$$F_1 = \sum_{i=1}^{n} \alpha |B_i| \qquad \text{s.t.} \quad \begin{cases} 0 \leq F_1 \leq F_{1m} \\ 0 \leq B_i \leq B_m \end{cases} \quad (7)$$

where: $F_1$ is the total required investment; $F_{1m}$ is the maximum amount available for investment; $B_i$ is the compensation at busbar *i* measured in MVAr; $B_m$ is the absolute value of the maximum amount of compensation in MVAr allowed at a single busbar of the system; $\alpha$ is the cost per MVAr of a capacitor bank and *n* is the number of busbars in the electric power system.

$F_2$: *Active power losses*

$$F_2 = P_g - P_l \geq 0 \quad (8)$$

where: $F_2$ is the total transmission active losses of the power system in MW; $P_g$ is the total active power generated in MW and $P_l$ is the total load of the system in MW.

$F_3$: *Average voltage deviation*

$$F_3 = \frac{\sum_{i=1}^{n} |V_i - V_i^*|}{n} \quad (9)$$

where: $F_3$ is the per unit (pu) average voltage difference; $V_i$ is the actual voltage at busbar *i* (pu) and $V_i$* is the desired voltage at busbar *i* (pu).

$F_4$: *Maximum voltage deviation*

$$F_4 = \max_i |V_i - V_i^*| = \|\mathbf{V} - \mathbf{V}^*\|_\infty \geq 0 \quad (10)$$

where $F_4$ is the maximum voltage deviation from the desired value (pu); $\mathbf{V} \in \Re^n$ is the voltage vector (unknown) and $\mathbf{V}^* \in \Re^n$ is the desired voltage vector.

In summary, the optimization problem to be solved is the following:

$$minimize \;\; \mathbf{F} = [F_1 \quad F_2 \quad F_3 \quad F_4] \tag{11}$$

where

$$\mathbf{F} = \begin{bmatrix} \sum_{i=1}^{n} B_i & P_g - P_l & \frac{\sum_{i=1}^{n} \left| V_i - V_i^* \right|}{n} & \left\| \mathbf{V} - \mathbf{V}^* \right\|_\infty \end{bmatrix}$$

subject to $0 \le F_1 \le F_{1m}$, $0 \le B_i \le B_m$ and the load flow equations [9]:

$$\begin{aligned} P_k &= V_k \sum_{i=1}^{n} Y_{ki} V_i \cos(\delta_k - \delta_i - \theta_{ki}), \\ Q_k &= V_k \sum_{i=1}^{n} Y_{ki} V_i sin(\delta_k - \delta_i - \theta_{ki}) \end{aligned} \tag{12}$$

where: $V_k$ is the voltage magnitude at node $k$; $Y_{ki}$ is the admitance matrix entry corresponding to nodes $k$ and $i$; $\delta_k$ is the voltage phase angle at node $k$; $\theta_{ki}$ is the phase admitance matrix entry corresponding to nodes $k$ and $i$; $P_k$ is the active power injected at node $k$; $Q_k$ is the reactive power injected at node $k$.

To represent the amount of reactive compensation to be allocated at each busbar $i$, a decision vector **B** [7], is used to indicate the size of each reactive bank in the power system, i.e.:

$$\mathbf{B} = [B_1 \quad B_2 \quad \cdots \quad B_n],\, B_i \in \Re, |B_i| \le B_m \tag{13}$$

Note that the *true* Pareto Optimal Set (termed $P_{\text{true}}$), with its corresponding $PF_{\text{true}}$, are not completely known in practice without extensive calculation (computationally not feasible in most situations). Therefore, it would be normally enough for practical purposes to find a *known* Pareto Optimal Set, termed $P_{\text{known}}$, with its corresponding Pareto Front $PF_{\text{known}}$, close enough to the true optimal solution [5].

## IV. Proposed approach

A new approach based on the Strength Pareto Evolutionary Algorithm was developed for this work. This method, closely related to Genetic Algorithms [12] generates a stored *External Population* composed by the best known individuals **B** of a general evolutionary population. This external group of solutions conforms $P_{\text{known}}$, available at each moment of the computation, i.e., the best known approximation to $P_{\text{true}}$. The original SPEA evaluates an individual's fitness depending on the number of decision vectors it dominates in an ordinary evolutionary population.

SPEA preserves population diversity using Pareto dominance relationship and incorporating a clustering procedure in order to reduce the nondominated set without destroying its characteristics. In general, cluster analysis partitions a collection of $r$ elements into $g$ groups of relatively homogeneous elements, where $g < r$, selecting a representative individual for each of the $g$ clusters [8]. That way, a limited number (up to $g$ individuals) may be maintained in the external population, preserving the main characteristics of the Pareto Front [7].

An important issue with SPEA is its convergence property, assured by *Theorem 4* proved in [5], a characteristic not always present in other MOEAs. Consequently, the algorithm implemented for this work is based on the original SPEA [7], but differs from it in the following aspects:

- *Scaling.* A special heuristic method is used to improve the fitness calculation in order to discourage individuals electrically not well compensated in strategic busbars. The proposed heuristic privileges busbars with large number of branches and good voltage profile. This is accomplish with the following scaling method:

  a. For each busbar $i$ of a scheme (individual of a population), calculate a (penalization) factor $K_i$ using the following expression:

  $$K_i = \begin{cases} (V_i^* - V_i)\; l_i & \text{if } V_i < V_i^* \\ 0 & \text{if } V_i \ge V_i^* \end{cases}$$

  where $l_i$ is the number of branches connected to node $i$. $K_i = 0$ indicates that no reactive compensation is heuristically recommended for busbar $i$.

  b. Evaluate a penalization scaling constant $K$ for each scheme using:

  $$K = \sum_{i=1}^{n} K_i$$

  c. Scale fitness according to:

  $$scaled\_fitness = \begin{cases} fitness / K & \text{if } K > 0 \\ fitness & \text{if } K = 0 \end{cases}$$

- *Stop criterion.* Computation is halted after a maximum number of generations or when no new nondominated solution is found to dominate an individual of the external population for a given number $N_{stop}$ of successive generations.

The proposed approach may be summarized as follows:

1. Generate an initial population *Pop* and create an empty external nondominated set $P_{known}$.
2. Copy nondominated members of *Pop* to $P_{known}$..
3. Remove individuals within $P_{known}$ which are covered (dominated) by any member of $P_{known}$.
4. If the number of externally nondominated solutions in $P_{known}$ exceeds a given maximum *g*, clustering is applied in order to reduce the external population to a size *g*.
5. Calculate the fitness of each individual in *Pop* as well as in $P_{known}$ using scaled SPEA fitness assignment procedure.
6. Select individual from *Pop* + $P_{known}$ (multiset union) until the mating pool is filled. For this study, roulette wheel selection was used.
7. Apply crossover and mutation standard genetic operators.
8. Go to step 2 if stop criterion is not verified.

## V. Experimental environment

As a study case, the IEEE 118 Bus Power Flow Test Case has been selected [13]. In order to stress the original system, its active and reactive loads were incremented by 40%, turning the power network in an adequate candidate for reactive power compensation.

For comparison purposes, the Pareto set generated by the proposed approach has been compared to a Pareto set obtained by a team of specialized engineers using standard computational programs that are here called *heuristic method*.

For the experimental results presented in the following section, it has been assumed for simplicity that $\alpha = 1$, i.e., capacitor banks have unitary cost per MVAr. At the same time, $N_{stop}$ = 100 was experimentally chosen.

To be able to compare two different sets of solutions, an appropriate test suite metrics is used [5], because no single metric can entirely capture total MOEA performance, effectiveness and efficiency. The test suit comprises the following metrics:

1) *Overall Nondominated Vector Generation (N)*

$$N \overset{\Delta}{=} \left| PF_{known} \right|_c$$

where $\left| \cdot \right|_c$ denotes cardinality.

This metric indicates the number of solutions in $PF_{\text{known}}$. A good $PF_{known}$ set is expected to have a large number of individuals, in order to offer a wide variety of options to designers.

2) *Overall Nondominated Vector Generation Ratio* (*ONVGR*)

$$ONVGR \overset{\Delta}{=} \frac{N}{\left| PF_{true} \right|_c}$$

It denotes the ratio between the number of solutions in $PF_{known}$ to the number of solutions in $PF_{true}$. Since the objective is to obtain a $PF_{known}$ set as similar as possible to $PF_{true}$, a value near to 1 is desired.

3) *Error Ratio* (*E*)

$$E \overset{\Delta}{=} \frac{\sum_{i=1}^{N} e_i}{N};$$

$$e_i = \begin{cases} 0 & \text{if a vector in } PF_{known} \text{ is also in } PF_{true} \\ 1 & \text{otherwise} \end{cases}$$

This ratio reports the proportion of objective vectors in $PF_{known}$ that are not members of $PF_{true}$. Therefore, an Error Ratio *E* close to 1 indicates a poor correspondence between $PF_{known}$ and $PF_{true}$, i.e., *E* = 0 is desired.

4) *Maximum Pareto Front Error* (*ME*)

$$ME \overset{\Delta}{=} \max_j \left( \min_i \left\| \mathbf{F}^i - \mathbf{F}^j \right\|_\infty \right)$$

$$\mathbf{F}^i \in PF_{true} \;;\; \mathbf{F}^j \in PF_{known}$$

It indicates the maximum error band that, when considered with respect to $PF_{known,}$ encompasses every vector in $PF_{true}$. Ideally, *ME* = 0 is desired.

5) *Generational Distance (G)*

$$G \overset{\Delta}{=} \frac{\left( \sum_{i=1}^{N} d_i^2 \right)^{1/2}}{N}$$

where $d_i$ is the Euclidean distance (in objective space) between each objective vector **F** in $PF_{known}$ and its nearest member in $PF_{true}$. A large value of *G* indicates $PF_{known}$ is far from $PF_{true}$, being *G* = 0 the ideal situation.

Since most of these metrics reflect the likeness between the true Pareto Front Optimal set $PF_{true}$ and a computed Pareto Front set $PF_{known}$, a good approximation of $PF_{true}$ is built from a complete set of solutions extensively calculated during several months.

## VI. Experimental results

For experimental purposes, a classical implementation of the SPEA MOEA was first tested, but it soon reached a stagnant population; i.e., no new solutions were obtained with new generations for $N_{stop}$ = 100 generations, satisfying the stop criterion. On the other hand, the proposed approach stopped using a maximum number of generation criterion, since it continues generating new solutions, not showing the premature convergence seen in our SPEA implementation. This is an important advantage of the new approach since it gives the user a wider variety of alternative solutions; therefore, only the proposed approach will be compared to the specialists' results. Consequently, the following tables presents experimental results using the IEEE-118 study case, comparing the solutions obtained with a typical run of the proposed approach with respect to the best solutions obtained by a team of highly specialized engineers using traditional computing tools.

Table I shows solutions obtained with the proposed approach while Table II does the same for the solution set generated by the specialists (heuristic method). Both tables present in the first column an ID for identification of each individual in the final Pareto set. Columns 2 to 5 contain the objective values obtained for each individual. The last column tells whether the individual is dominated or not by any solution in the other set.

TABLE I
Proposed approach: performance of solutions

| # | $F_1$ | $F_2$ | $F_3$ | $F_4$ | *Dominated by* |
|---|---|---|---|---|---|
| $SP_1$ | 133.14 | 472 | 0.0492 | 0.0096 | |
| $SP_2$ | 133.35 | 454 | 0.0494 | 0.0099 | |
| $SP_3$ | 133.52 | 498 | 0.0488 | 0.0101 | $H_1$ |
| $SP_4$ | 132.59 | 497 | 0.0485 | 0.0101 | |
| $SP_5$ | 132.47 | 486 | 0.0485 | 0.0102 | |
| $SP_6$ | 132.55 | 484 | 0.0499 | 0.0105 | |
| $SP_7$ | 132.78 | 473 | 0.0465 | 0.0111 | |
| $SP_8$ | 133.48 | 360 | 0.0496 | 0.0111 | |
| $SP_9$ | 132.78 | 294 | 0.0490 | 0.0116 | |
| $SP_{10}$ | 134.66 | 424 | 0.0493 | 0.0117 | $H_4$ |
| $SP_{11}$ | 132.80 | 424 | 0.0500 | 0.0117 | |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | |
| $SP_{265}$ | 132.82 | 459 | 0.0458 | 0.0117 | |

TABLE II
Heuristic method: performance of solutions

| # | $F_1$ | $F_2$ | $F_3$ | $F_4$ | *Dominated by* |
|---|---|---|---|---|---|
| $H_1$ | 133.07 | 497 | 0.0476 | 0.0098 | |
| $H_2$ | 133.08 | 495 | 0.0503 | 0.0098 | |
| $H_3$ | 133.14 | 472 | 0.0507 | 0.0101 | $SP_1$ |
| $H_4$ | 134.58 | 406 | 0.0486 | 0.0110 | |
| $H_5$ | 132.72 | 499 | 0.0510 | 0.0115 | $SP_6$ |
| $H_6$ | 132.75 | 491 | 0.0513 | 0.0116 | $SP_4$ |
| $H_7$ | 132.78 | 487 | 0.0515 | 0.0116 | $SP_7$ |
| $H_8$ | 132.79 | 481 | 0.0515 | 0.0116 | $SP_9$ |
| $H_9$ | 132.86 | 473 | 0.0517 | 0.0117 | $SP_{11}$ |
| $H_{10}$ | 132.91 | 466 | 0.0518 | 0.0118 | $SP_{11}$ |
| $H_{11}$ | 132.93 | 463 | 0.0518 | 0.0119 | $SP_{265}$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | |
| $H_{170}$ | 135.00 | 162 | 0.0522 | 0.0131 | |

As shown in the last columns of tables I and II, there are more individuals calculated by the proposed approach that dominate solutions given by the specialists than the other way around. Clearly, the proposed method overcome the heuristic one in the quality of the solutions. Additionally, the proposed method has the advantage of presenting a wider variety of options (265 solutions) with fewer highly specialize engineers.

Table III shows the experimental results of a comparison based on the test suite metrics presented in section V. As previously remarked, the proposed metrics try to measure the similarity between the solutions set and the True Pareto set in objective space. From the experimental results, it can be confirmed that the proposed approach offers better solutions than the ones proposed by the experts, as clearly indicated by the first four metrics, and noting that the relative difference in *Generational Distance* metric ($G$) is relatively small (less than 3.2 %). In fact, the proposed approach presents a larger set of solutions ($N$ = 265 vs. 232) that are mostly true solutions (*ONVGR* of 86,6 % vs. 75.8 % for the heuristic method) and with smaller error ($E$ = 0.076 and $ME$ = 0.048).

TABLE III
Results using Comparison Metrics

| *Metric* | *Proposed approach* | *Heuristic method* | *Relative difference* *100*(prop. - heu.) / prop.* | *Best method* |
|---|---|---|---|---|
| $N$ | 265 | 232 | 12.453 % | Proposed |
| *ONVGR* | 0.866 | 0.758 | 12.471 % | Proposed |
| $E$ | 0.076 | 0.302 | -299.603 % | Proposed |
| $ME$ | 0.048 | 0.078 | -62.343 % | Proposed |
| $G$ | 0.609 | 0.590 | 3.118 % | Heuristic |

## VII. Concluding remarks

In this paper, Reactive Compensation Problem is first treated as a *Multi-objective Optimization Problem* with 4 conflicting objective functions: (i) investment in reactive compensation devices, (ii) active power losses, (iii) average voltage deviation and (iv) maximum voltage deviation.

To solve the problem, a new approach based on *SPEA* is proposed. This new approach introduces new features such as: (i) a fitness scaling technique and (ii) a stop criterion.

For comparison purposes, the solution set obtained in a single run of the proposed approach is compared to a set of heuristic schemes elaborated by a team of specialists.

Experimental results using the proposed approach demonstrated several advantages when using the proposed method, such as: (i) a set of solutions closer to the *True Pareto Set* outperforming the heuristic approach in most of the studied figures of merits, (ii) highly reduced need for specialized human resources due to the automatic nature of the method and (iii) a wider variety of options. This last feature is of special importance, since a richer set of alternatives is offered to the network planners. In order to select sub-sets of solutions which best fit the interests of the user, an adaptive constrain philosophy is suggested. That way, the network engineer may restrict the constraints to reduce the number of solutions after having a good idea of the whole Pareto solutions, searching forward only in the redefined domain. This process may continue iteratively until a good solution with an acceptable compromise among objective functions is found [9].

As future work, new specialized (genetic and/or heuristic) operators are being developed to locally improve reactive compensation of a given individual. At the same time, other objective functions (such as voltage stability margin) are going to be studied. Finally, parallel asynchronous computation using a network of computers is considered for larger networks with more objective functions, given the huge need of resources in order to optimize investments and energy transmission in large real world systems.

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