Multi-objective Optimization in Reactive Power Compensation.

José Vallejos. *jvallejo@cnc.una.py*
Rodrigo Ramos. *rodrigo.ramos@mailexcite.com*
Benjamín Barán. *bbaran@cnc.una.py*
Centro Nacional de Computación, Universidad Nacional de Asunción.
Campus Universitario, San Lorenzo, Paraguay.
Teléfono: +595 21 585 550; Fax:+595 21 585 619

Abstract.

This paper presents a new approach to treat Reactive Power Compensation in Power Systems using a Multi-objective Optimization Evolutionary Algorithm. A variant of the Strength Pareto Evolutionary Algorithm is proposed to independently optimize several parameters instead of traditional constrained Single-objective approach where an objective function is a linear combination of several factors, such as, investment and transmission losses, with several constrains that limit other parameters as reliability and voltage profile.

At the same time, computational results are presented showing that a good amount of practical solutions are found with the proposed method instead of a set of compensation schemes elaborated by a team of specialized engineers specialists using nowadays methods. Comparison results using appropriate test suit metrics emphasize outstanding advantages of the proposed computational approach, such as: ease of calculation, better defined Pareto Front and a larger number of Pareto solutions.

1. Introduction.

Reactive Power Compensation (RPC) in power systems is a very important issue in the expansion planning and operation of power systems because it leads to increased transmission capability, reduced losses and improved power factor using shunt capacitors that have been very commonly installed in transmission and distribution networks. By applying capacitors adjacent to loads, several advantages are obtained some of them are [1, 2]:

- improved power factor,
- reduced transmission losses,
- increased transmission capability,
- improved voltage control,
- improved power quality.

Achievement of these items depends mainly on an adequate allocation of shunt capacitor banks. Thus, this problem can be stated as the determination of the locations and the capacities of new sources of reactive power, searching simultaneously to accomplish the following goals:

- *a good bus tension profile*: the quality of service is directly related to the difference between the effective and the nominal bus voltage,
- *minimization of transmission losses*: active power transmission losses can be directly translated into monetary losses since they are the main component in the difference between the generated power and the consumed power,
- *minimization of the amount of reactive compensation installed*: although shunt capacitor compensation generally provides the most economical reactive power source for voltage control, heavy use of these devices could lead to the reduction of stability margins and poor voltage regulation [2].

RPC is commonly addressed as a constrained *Single-objective Optimization Problem* (SOP) [3-7]. In this context, the objective function is a linear combination of several factors, such as: investment and transmission losses, subject to operational constrains such as reliability and voltage profile [2]. Traditional Multi-objective Optimization Algorithm usually provide a unique optimal solution. On the contrary, *Multi-objective Optimization Evolutionary Algorithms* (MOEA) independently and simultaneously optimize several parameters turning most traditional constraints into new objective functions. This seems more natural for real world problems where choosing a threshold may seem arbitrary [8]. As a result, a wide set of optimal solutions (Pareto set) may be found. Therefore, an engineer may have a whole set of optimal alternatives before deciding which solution is the best compromise of different (and sometimes contradictory) features. This approach has already been treated as a *Multi-objective Optimization Problem* (MOP) with two conflicting objective functions [7].

To solve the RPC, this paper presents a new approach based on the *Strength Pareto Evolutionary Algorithm* (SPEA) [9]. SPEA shares the following characteristics with other MOEAs:

- stores the nondominated solutions found so far externally,
- uses the concept of Pareto dominance in order to assign scalar fitness value to the individuals,
- performs clustering to reduce the number of nondominated solutions stored without destroying the characteristics of the tradeoff front.

We have chosen SPEA because it has the following unique characteristics:

- it combines the techniques above exposed in a single algorithm,
- the fitness of an individual is determined only from the solutions stored in the external nondominated set, whether members of the population dominate each other is irrelevant,
- all the solutions that are included in the external nondominated population participate in the selection;
- it introduces clustering, a new method in order to induce niching [10,11],
- its convergence property, assured by *Theorem 4* proved in [8],
- solutions present, with respect to other MOEAs, a smaller distance to the Pareto-optimal front [12,13].

This main objective of this works is show the convenience of the SPEA algorithm in the search of good practical solutions, with compensation schemes that lead to interesting savings in infrastructure investment and operative performance of the power system.

2. Multi-objective Optimization Problems.

A general MOP[8] includes a set of n decision variables, a set of *k* objective functions, and a set of *m* restrictions. Objective functions and restrictions are functions of decision variables. This can be expressed mathematically as:

$$\begin{array}{ll} \textit{Optimize} & \mathbf{y} = \mathbf{F}(\mathbf{x}) = [F_1(\mathbf{x}) \quad F_2(\mathbf{x}) \quad \cdots \quad F_k(\mathbf{x})] \\ \textit{s.t.} & \mathbf{e}(\mathbf{x}) = [e_1(\mathbf{x}) \quad e_2(\mathbf{x}) \quad \cdots \quad e_m(\mathbf{x})] \geq \mathbf{0} \\ \textit{where} & \mathbf{x} = [x_1 \quad x_2 \quad \cdots \quad x_n] \in X \\ & \mathbf{y} = [y_1 \quad y_2 \quad \cdots \quad y_k] \in Y \end{array} \tag{1}$$

$\mathbf{x}$ is known as decision vector and $\mathbf{y}$ as objective vector. X denotes the decision space and the objective space is denoted by Y. Depending on the problem at hand "optimize" could mean minimize or maximize.

The set of restrictions $\mathbf{e}(\mathbf{x}) \geq \mathbf{0}$ determines the set of feasible solutions X_f and its corresponding set of feasible objective vectors Y_f.

From this definition, it follows that every solution consists of a n-tuple $\mathbf{x}$, that yields an objective vector $\mathbf{y}$, where every $\mathbf{x}$ must satisfy the set of restrictions $\mathbf{e}(\mathbf{x}) \geq \mathbf{0}$. The optimization problem consists in finding the $\mathbf{x}$ that has the "best" $\mathbf{F}(\mathbf{x})$. In general, there is not one "best" solution, but a set of solutions, none of which can be considered better than the others if all objectives are considered at the same time. This derives from the fact that there could be (and mostly there are) conflicts between the different objectives that compose a problem. Thus, a new concept of optimality should be established for MOPs.

In common SOPs the set of feasible decision variables is completely ordered by the objective function F. The goal is simply to find the value (or set of values) that lead to the optimal values of F. In contrast, in multi-objective optimization the feasible decision vector set is only partially ordered; i.e., there exist a decision vector $\mathbf{x_1}$ and a decision vector $\mathbf{x_2}$ and $\mathbf{F}(\mathbf{x_1})$ cannot be considered better than $\mathbf{F}(\mathbf{x_2})$, neither $\mathbf{F}(\mathbf{x_2})$ is better than $\mathbf{F}(\mathbf{x_1})$. Then, mathematically the relations =, $\leq$ and $\geq$ should be extended. This could be done using the concept of *dominance* as explained below. In fact, given two decision vectors $\mathbf{u}, \mathbf{v} \in X$ in a context of minimization, we can state:

$$\begin{array}{llll} \mathbf{F}(\mathbf{u}) = \mathbf{F}(\mathbf{v}) & \textit{iff} & F_i(\mathbf{u}) = F_i(\mathbf{v}) & \forall i \in \{1, 2, \ldots, k\} \\ \mathbf{F}(\mathbf{u}) \leq \mathbf{F}(\mathbf{v}) & \textit{iff} & F_i(\mathbf{u}) \leq F_i(\mathbf{v}) & \forall i \in \{1, 2, \ldots, k\} \\ \mathbf{F}(\mathbf{u}) < \mathbf{F}(\mathbf{v}) & \textit{iff} & \mathbf{F}(\mathbf{u}) \leq \mathbf{F}(\mathbf{v}) \wedge \mathbf{F}(\mathbf{u}) \neq \mathbf{F}(\mathbf{v}); & \end{array} \quad (2)$$

where $\wedge$ denotes an *and* operation.

The relations $\geq$ and $>$ could be defined in similar ways. Then, given two decision vectors of a MOP, $\mathbf{x_1}$ and $\mathbf{x_2}$ they comply to one of three possible conditions:

- either $\mathbf{F}(\mathbf{x}_1) < \mathbf{F}(\mathbf{x}_2)$,
- or $\mathbf{F}(\mathbf{x}_2) < \mathbf{F}(\mathbf{x}_1)$,
- or $\mathbf{F}(\mathbf{x}_1) \nleq \mathbf{F}(\mathbf{x}_2) \wedge \mathbf{F}(\mathbf{x}_2) \nleq \mathbf{F}(\mathbf{x}_1)$.

The above relations may be expressed with the following symbols:

Pareto Dominance. Given two objective vectors $\mathbf{a}, \mathbf{b} \in X$

$$\begin{array}{lll} \mathbf{a} \succ \mathbf{b} \ (\mathbf{a} \textit{ dominates } \mathbf{b}) & \textit{iff} & \mathbf{a} < \mathbf{b} \\ \mathbf{b} \succ \mathbf{a} \ (\mathbf{b} \textit{ dominates } \mathbf{a}) & \textit{iff} & \mathbf{b} < \mathbf{a} \\ \mathbf{a} \sim \mathbf{b} \ (\mathbf{a} \textit{ and } \mathbf{b} \textit{ are not comparable}) & \textit{iff} & \mathbf{a} \nleq \mathbf{b} \wedge \mathbf{b} \nleq \mathbf{a} \end{array} \quad (3)$$

Definitions for the maximization and maximization/minimization problems could be formulated in a similar way.

At this point the concept of Pareto optimality can be introduced. A solution is said to be Pareto-optimal or “non inferior” if any objective can not be improved without degrading others.

Pareto Optimality. A decision vector $\mathbf{x} \in X_f$ and its corresponding objective vector $\mathbf{y} = \mathbf{F}(\mathbf{x}) \in Y_f$ is non-dominated with respect to a set $A \subseteq X_f$ if and only if

$$\forall\ a \in A : (\mathbf{F}(\mathbf{x}) \succ \mathbf{F}(\mathbf{a}) \vee \mathbf{F}(\mathbf{x}) \sim \mathbf{F}(\mathbf{a})) \quad (4)$$

where $\vee$ denotes an *or* operation.

When $\mathbf{x}$ is non-dominated with respect to the whole set X_f (and only in this case) $\mathbf{x}$ is a *Pareto-optimal solution.* The whole set of Pareto-optimal solutions is known as Pareto-optimal set P; i.e.

$$P = \{\ \mathbf{x} \in X_f \mid \mathbf{F}(\mathbf{x}) \succ \mathbf{F}(\mathbf{v}) \vee \mathbf{F}(\mathbf{x}) \sim \mathbf{F}(\mathbf{v}) \quad \forall\ \mathbf{v} \in X_f \} \quad (5)$$

The corresponding set of objective vectors $\mathbf{y}$ is known as *Pareto-optimal front FP*; i.e.,

$$FP = \{\ \mathbf{y} \in Y_f \mid \mathbf{y} = \mathbf{F}(\mathbf{x}) \quad \forall\ \mathbf{x} \in P\ \} \quad (6)$$

Dealing with Pareto-optimal solutions, it is clear that they are non-comparable. This points to the fact that a MOP does not always have a single solution, but a set of compromise solutions. None of these solutions can be defined as “the best”, unless other information is added (as a weight for every objective).

3. Mathematical formulation.

For the purposes of this paper, the following assumptions are considered in the formulation of the problem:

- shunt-capacitor bank cost per MVAr is the same for all busbars of the power system,
- power system is considered only at peak load.

Based on these considerations [2, 14], four objective functions F_i (to be minimized) have been identified for the present work: F_1 and F_2 are related to investment and transmission losses, while F_3 and F_4 are related to quality of service. The objective functions to be considered are:

3.1. F_1: Investment in reactive compensation devices.

$$F_1 = \sum_{i=1}^{n} \alpha \left| B_i \right| \qquad \text{s.t.} \quad \begin{cases} 0 \le \mathrm{F}_1 \le F_{1m} \\ 0 \le B_i \le B_m \end{cases}, \tag{7}$$

where: F_1 is the total required investment; F_{1m} is the maximum amount available for investment; B_i is the compensation at busbar i measured in MVAr; B_m is the absolute value of the maximum amount of compensation in MVAr allowed at a single busbar of the system; α is the cost per MVAr of a capacitor bank and n is the number of busbars in the electric power system.

3.2. F_2: Active power losses.

$$F_2 = P_g - P_l \ge 0, \tag{8}$$

where: F_2 is the total transmission active losses of the power system in MW; P_g is the total active power generated in MW and P_l is the total load of the system in MW.

3.3. F_3: Average voltage deviation.

$$F_3 = \frac{\sum_{i=1}^{n} \left| V_i - V_i^* \right|}{n}, \tag{9}$$

where: F_3 is the per unit (pu) average voltage difference; V_i is the actual voltage at busbar i (pu) and V_i* is the desired voltage at busbar i (pu).

3.4. F_4: Maximum voltage deviation.

$$F_4 = \max_i \left| V_i - V_i^* \right| = \left\| \mathbf{V} - \mathbf{V}^* \right\|_\infty \ge 0, \tag{10}$$

where F_4 is the maximum voltage deviation from the desired value (pu); $\mathbf{V} \in \Re^n$ is the voltage vector (unknown) and $\mathbf{V}^* \in \Re^n$ is the desired voltage vector.

In summary, the optimization problem to be solved is the following:

$$\mathit{minimize} \;\; \mathbf{F} = \left[F_1 \quad F_2 \quad F_3 \quad F_4 \right], \tag{11}$$

where

$$\mathbf{F} = \left[\sum_{i=1}^{n} B_i \quad P_g - P_l \quad \frac{\sum_{i=1}^{n} \left| V_i - V_i^* \right|}{n} \quad \left\| \mathbf{V} - \mathbf{V}^* \right\|_\infty \right],$$

subject to $0 \le \mathrm{F}_1 \le F_{1m}$, $0 \le B_i \le B_m$ and the load flow equations [15]:

$$P_k = V_k \sum_{i=1}^{n} Y_{ki} V_i \cos(\delta_k - \delta_i - \theta_{ki}),$$
$$Q_k = V_k \sum_{i=1}^{n} Y_{ki} V_i sin(\delta_k - \delta_i - \theta_{ki}), \qquad (12)$$

where: V_k is the voltage magnitude at node k; Y_{ki} is the admitance matrix entry corresponding to nodes k and i; δ_k is the voltage phase angle at node k; θ_{ki} is the phase admitance matrix entry corresponding to nodes k and i; P_k is the active power injected at node k; Q_k is the reactive power injected at node k.

To represent the amount of reactive compensation to be allocated at each busbar i, a decision vector **B** [9], is used to indicate the size of each reactive bank in the power system, i.e.:

$$\mathbf{B} = [B_1 \quad B_2 \quad \cdots \quad B_n], B_i \in \Re, |B_i| \le B_m, \qquad (13)$$

Note that the *true* Pareto-optimal set (termed P_{true}), with its corresponding PF_{true}, are not completely known in practice without extensive calculation (computationally not feasible in most situations). Therefore, it would be normally enough for practical purposes to find a *known* Pareto-optimal set, termed P_{known}, with its corresponding Pareto Front PF_{known}, close enough to the true optimal solution [8].

4. Proposed approach.

A new approach based on the Strength Pareto Evolutionary Algorithm was developed for this work, but differs from [9] and [16] in the following aspects:

- *Encoding.* The location and size of shunt capacitors are encoded by a vector of integer numbers where each component corresponds to a busbar of the electric power system and the value of each component represents the quantity of 1 MVAr capacitors in the bank of shunt capacitors, given that the size of these devices is in fact quantisized in commercial applications. A non binary alphabet allows to consider different sizes of shunt capacitors easily [17]. An individual is thus represented by:

  $$\mathbf{x} = [x_1 \quad x_2 \quad \cdots \quad x_n], \qquad x_i \in \{0,1,2,\ldots,B_m\}, \qquad (14)$$

  where $\mathbf{x}$ is an individual; x_i is the size of the shunt capacitor bank allocated at busbar *i; and n* is the number of busbars in the electric power system. As an example, figure 1 presents an electrical power system with three busbars (n=3) with a compensation vector $\mathbf{x}$ = [12 4 7].

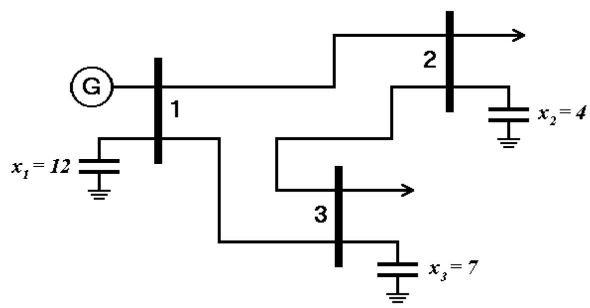

Figure 1. Example of a power system.

- *Heuristic Initialization.* A special heuristic method is used to generate the initial population in order to obtain individuals electrically well compensated. The proposed heuristic is based on encouraging compensation at busbars with large number of branches and voltage profile far from the desired value. This is done by using a method summarized as follows:
  1. Choose a total amount of compensation B_{tot}.
  2. For each busbar i of the system, calculate a factor H_i using the following expression:

$$H_i = \begin{cases} (V_i - V_i^*)\, l_i & \text{if } V_i < V_i^* \\ 0 & \text{if } V_i \geq V_i^* \end{cases}$$

where l_i is the number of branches connected to node i. $H_i = 0$ indicates that no reactive compensation is heuristically assigned to busbar i.

3. Normalize H_i using:

$$H_i^{'} = \frac{H_i}{\sum_{i=1}^{n} H_i},$$

4. Compensate each busbar i with B_i calculated as follows:

$$B_i = H_i^{'} B_{tot},$$

- *Scaling.* A special heuristic method is used to improve the fitness calculation in order to discourage individuals electrically not well compensated in strategic busbars. The proposed heuristic privileges busbars with large number of branches and good voltage profile. This is accomplish with the following scaling method:
  1. For each busbar i of a scheme (individual of a population), calculate a (penalization) factor K_i using the following expression:

  $$K_i = \begin{cases} (V_i^* - V_i)\, l_i & \text{if } V_i < V_i^* \\ 0 & \text{if } V_i \geq V_i^* \end{cases}$$

  where l_i is the number of branches connected to node i. $K_i = 0$ indicates that no reactive compensation is heuristically recommended for busbar i.
  2. Evaluate a penalization scaling constant K for each scheme using:

  $$K = \sum_{i=1}^{n} K_i,$$

  3. Scale fitness according to:

  $$scaled\_fitness = \begin{cases} fitness/K & \text{if } K > 0 \\ fitness & \text{if } K = 0 \end{cases}$$

- *Stop criterion.* Computation is halted after a maximum number of generations or when no new nondominated solution is found to dominate an individual of the external population for a given number *Nstop* of successive generations.

- *Two External Populations*. If only one external population is used, it is possible*:*
  1. to save all found Pareto solutions, but this population may become too large and the evolutionary population looses genetic importance in the search process, or
  2. to loose found solutions using clustering to maintain a given number g of external solutions (original SPEA approach).

  In this new proposal, two external populations are stored, one with all found nondominated solutions and another with a maximum number *g* of nondominated individuals, fixed by clustering, that participates in the ordinary evolutionary process. That way, the external population used in the evolutionary process does not diminish the influence of the evolutionary population and no optimal solution is lost. Note that this second external population does not participate in the evolutionary process.

The proposed approach may be summarized as follows:

1. Generate an initial population *Pop* with heuristic initialization procedure and create an empty external nondominated set P_{known}.
2. Copy nondominated members of *Pop* to P_{known}..Remove individuals within P_{known} which are covered (dominated) by any member of P_{known}.

3. If the number of externally nondominated solutions in P_{known} exceeds a given maximum g, clustering is applied in order to reduce the external population to a size g.
4. Calculate the fitness of each individual in *Pop* as well as in P_{known} using scaled SPEA fitness assignment procedure.
5. Select individual from $Pop + P_{known}$ (multiset union) until the mating pool is filled. For this study, roulette wheel selection was used.
6. Apply crossover and mutation standard genetic operators.
7. Go to step 2 if stop criterion is not verified.

5. Experimental environment.

As a study case, the IEEE 118 Bus Power Flow Test Case has been selected [18]. In order to stress the original system, its active and reactive loads were incremented by 40%, turning the power network in an adequate candidate for RPC. This IEEE 118 Bus Test Case represents a portion of the American Electric Power Systems (in the Midwestern US) as of December, 1962. A schematich representation of this system is shown in figure 2.

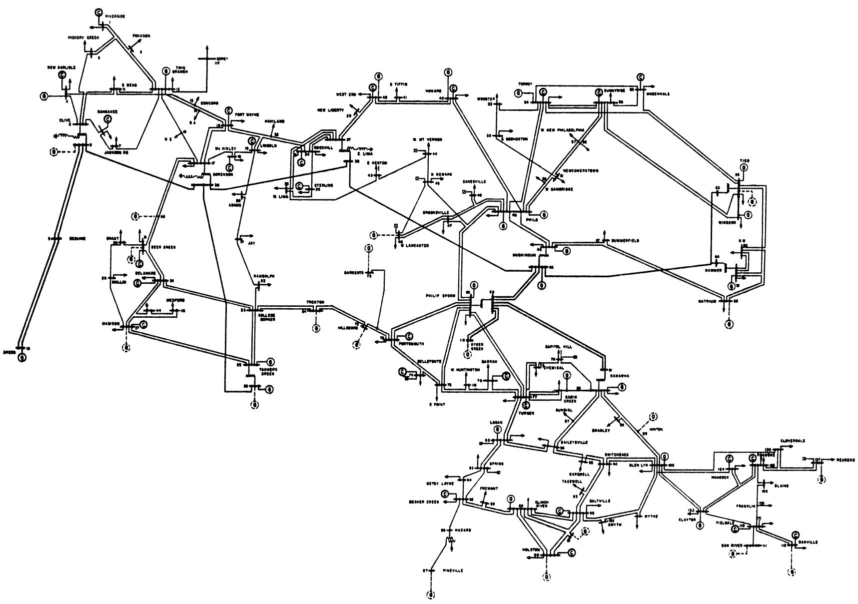

Figure 2. IEEE 118 Bus Power Flow Test Case.

The computational platform used was a PC with a 350 MHz processor, and 128 MB RAM. The codification has been implemented in *MATLAB 5.3*[1], using *MATFLOW*[2] routine to solve each *load flow* for fitness calculation.

For comparison purposes, the Pareto set generated by the proposed approach has been compared to a Pareto set obtained by a team of specialized engineers using standard computational programs that will be called *heuristic method* for this work.

For the experimental results presented in the following section, it has been assumed for simplicity that $\alpha = 1$, i.e., capacitor banks have unitary cost per MVAr. At the same time, N_{stop} = 100 was experimentally chosen.

To be able to compare two different sets of solutions, an appropriate test suite metrics is used [8], because no single metric can entirely capture total MOEA performance, effectiveness and efficiency. The test suit comprises the following metrics:

[1] Copyright 1984-1999 by The Math Works, Inc.
[2] F. CARDOZO AND U. FERNÁNDEZ {fcardozo, ufernandez}@ieee.org

5.1. Overall Nondominated Vector Generation (*N*).

$$N \stackrel{\Delta}{=} \left|PF_{known}\right|_c ,$$

where $\left|\cdot\right|_c$ denotes cardinality.

This metric indicates the number of solutions in PF_{known}. A good PF_{known} set is expected to have a large number of individuals, in order to offer a wide variety of options to designers.

5.2. Overall Nondominated Vector Generation Ratio (*ONVGR*).

$$ONVGR \stackrel{\Delta}{=} \frac{N}{\left|PF_{true}\right|_c} ,$$

It denotes the ratio between the number of solutions in PF_{known} to the number of solutions in PF_{true}. Since the objective is to obtain a PF_{known} set as similar as possible to PF_{true}, a value near to 1 is desired.

5.3. Error Ratio (*E*).

$$E \stackrel{\Delta}{=} \frac{\sum_{i=1}^{N} e_i}{N} ,$$

$$e_i = \begin{cases} 0 & \text{if a vector in } PF_{known} \text{ is also in } PF_{true} \\ 1 & \text{otherwise} \end{cases}$$

This ratio reports the proportion of objective vectors in PF_{known} that are not members of PF_{true}. Therefore, an Error Ratio E close to 1 indicates a poor correspondence between PF_{known} and PF_{true}, i.e., $E = 0$ is desired.

5.4. Maximum Pareto Front Error (*ME*).

$$ME \stackrel{\Delta}{=} \max_j \left(\min_i \left\| \mathbf{F}^i - \mathbf{F}^j \right\|_\infty \right),$$

$$\mathbf{F}^i \in PF_{true} \;\; ; \;\; \mathbf{F}^j \in PF_{known}$$

It indicates the maximum error band that, when considered with respect to PF_{known}, encompasses every vector in PF_{true}. Ideally, $ME = 0$ is desired.

5.5. Generational Distance (*G*).

$$G \stackrel{\Delta}{=} \frac{\left(\sum_{i=1}^{N} d_i^2\right)^{1/2}}{N} ,$$

where d_i is the Euclidean distance (in objective space) between each objective vector **F** in PF_{known} and its nearest member in PF_{true}. A large value of G indicates PF_{known} is far from PF_{true}, being $G = 0$ the ideal situation.

Since most of these metrics reflect the likeness between the true Pareto Front Optimal set PF_{true} and a computed Pareto Front set PF_{known}, a good approximation of PF_{true} is built from a complete set of solutions extensively calculated during several months.

6. Experimental results.

For experimental purposes, a classical implementation of the SPEA MOEA was first tested, but it soon reached a stagnant population; i.e., no new solutions were obtained with new generations for N_{stop} = 100 generations, satisfying the stop criterion. On the other hand, the proposed approach stopped using a maximum number of generation criterion, since it continues generating new solutions, not showing the premature convergence seen in our SPEA implementation. This is an important advantage of the new approach since it gives the user a wider variety of alternative solutions; therefore, only the proposed approach will be compared to the specialists' results. Consequently, the following tables presents experimental results using the IEEE-118 study case, comparing the solutions obtained with a typical run of the proposed approach with respect to the best solutions obtained by a team of highly specialized engineers using traditional computing tools.

Table 1 shows solutions obtained with the proposed approach while table 2 does the same for the solution set generated by the specialists (heuristic method). Both tables present in the first column an ID for identification of each individual in the final Pareto set. Columns 2 to 5 contain the objective values obtained for each individual. The last column tells whether the individual is dominated (or not) by any solution in the other set.

Table 1. Proposed approach: performance of solutions.

#	F_1	F_2	F_3	F_4	*Dominated by*
SP_1	133.14	472	0.0492	0.0096	
SP_2	133.35	454	0.0494	0.0099	
SP_3	133.52	498	0.0488	0.0101	H_1
SP_4	132.59	497	0.0485	0.0101	
SP_5	132.47	486	0.0485	0.0102	
SP_6	132.55	484	0.0499	0.0105	
SP_7	132.78	473	0.0465	0.0111	
SP_8	133.48	360	0.0496	0.0111	
SP_9	132.78	294	0.0490	0.0116	
SP_{10}	134.66	424	0.0493	0.0117	H_4
SP_{11}	132.80	424	0.0500	0.0117	
⋮	⋮	⋮	⋮	⋮	
SP_{265}	132.82	459	0.0458	0.0117	

Table 2. Heuristic method: performance of solutions.

#	F_1	F_2	F_3	F_4	*Dominated by*
H_1	133.07	497	0.0476	0.0098	
H_2	133.08	495	0.0503	0.0098	
H_3	133.14	472	0.0507	0.0101	SP_1
H_4	134.58	406	0.0486	0.0110	
H_5	132.72	499	0.0510	0.0115	SP_6
H_6	132.75	491	0.0513	0.0116	SP_4
H_7	132.78	487	0.0515	0.0116	SP_7
H_8	132.79	481	0.0515	0.0116	SP_9
H_9	132.86	473	0.0517	0.0117	SP_{11}
H_{10}	132.91	466	0.0518	0.0118	SP_{11}
H_{11}	132.93	463	0.0518	0.0119	SP_{265}
⋮	⋮	⋮	⋮	⋮	
H_{170}	135.00	162	0.0522	0.0131	

As shown in the last columns of tables 1 and 2, there are more individuals calculated by the proposed approach that dominate solutions given by the specialists than the other way around. Clearly, the proposed method overcome the heuristic one in the quality of the solutions. Additionally, the proposed method has the advantage of presenting a wider variety of options (265 solutions) with fewer highly specialize engineers.

Table 3 shows the experimental results of a comparison based on the test suite metrics presented in section 5. As previously remarked, the proposed metrics try to measure the similarity between the solutions set and the True Pareto set in objective space. From the experimental results, it can be confirmed that the proposed approach offers better solutions than the ones proposed by the experts, as clearly indicated by the first four metrics, and noting that the relative difference in *Generational Distance* metric (*G*) is relatively small (less than 3.2 %). In fact, the proposed approach presents a larger set of solutions (*N* = 265 vs. 232) that are mostly true solutions (*ONVGR* of 86,6 % vs. 75.8 % for the heuristic method) and with smaller error (*E* = 0.076 and *ME* = 0.048).

Tabla 3. Results using Comparison Metrics.

Metric	***Proposed approach***	***Heuristic method***	***Relative difference*** $\frac{100*(prop. - heu.)}{prop.}$	***Best method***
N	265	232	12.453 %	Proposed
ONVGR	0.866	0.758	12.471 %	Proposed
E	0.076	0.302	-299.603 %	Proposed
ME	0.048	0.078	-62.343 %	Proposed
G	0.609	0.590	3.118 %	Heuristic

7. Concluding remarks.

In this paper, Reactive Compensation Problem is treated as a Multi-objective Optimization Problem with four conflicting objective functions:

(i) investment in reactive compensation devices,
(ii) active power losses,
(iii) average voltage deviation,
(iv) maximum voltage deviation.

To solve the problem, a new approach based on SPEA is proposed. This new approach introduces new features such as:

(i) a heuristic initialization,
(ii) a fitness scaling technique,
(iii) a stop criterion, and
(iv) two external populations.

For comparison purposes, the solution set obtained in a single run of the proposed approach is compared to a set of heuristic schemes elaborated by a team of specialists.

Experimental results using the proposed approach demonstrated several advantages, such as:

(i) a set of solutions closer to the *True Pareto Set* outperforming the heuristic approach in most of the studied figures of merits,
(ii) highly reduced need for specialized human resources due to the automatic nature of the method, and
(iii) a wider variety of options.

This last feature is of special importance, since a richer set of alternatives is offered to the network planners. In order to select sub-sets of solutions which best fit the interests of the user, an adaptive constrain philosophy is suggested. That way, the network engineer may restrict the constraints to reduce the number of solutions after having a good idea of the whole Pareto solutions, searching forward only in the redefined domain. This process may continue iteratively until a good solution with an acceptable compromise among objective functions is found [13].

As future work parallel asynchronous computation using a network of computers is considered for larger networks with more objective functions, given the huge need of resources in order to optimize investments and energy transmission in large real world systems.

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