Multi-objective Reactive Power Compensation

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***Abstract* — Reactive Power Compensation in Electric Systems is usually studied as a constrained Single-objective Optimization Problem where an objective function is a linear combination of several factors, such as, investment and transmission losses. At the same time, constrains limit other parameters as reliability and voltage profile.**

This paper presents a new approach using Multi-objective Optimization Evolutionary Algorithms. It proposes a variant of the Strength Pareto Evolutionary Algorithm (SPEA) that independently optimizes several parameters, turning most traditional constraints into new objective functions. That way, a wide set of optimal solutions, known as Pareto set, is found before deciding which solution best combines different features.

Several sets of solutions calculated by different methods are compared to a Pareto set found with the proposed approach using appropriate test suite metrics. Comparison results emphasize outstanding advantages of the proposed computational approach, such as: ease of calculation, better defined Pareto Front and a larger number of Pareto solutions.

***Index Terms* — Reactive Power Compensation, Multi-objective Optimization, Evolutionary Algorithms.**

I. Introduction

REACTIVE Power Compensation is commonly addressed as a constrained Single-objective Optimization Problem [1-3]. It basically consists in determining an adequate location and size of shunt capacitor/reactor banks. In this context, the objective function is a linear combination of several factors, such as: investment and transmission losses, subject to operational constrains such as reliability and voltage profile [4]. Traditional Single-objective Optimization Algorithms usually provide a unique optimal solution. On the contrary, *Multi-objective Optimization Evolutionary Algorithms* (MOEA) independently and simultaneously optimize several parameters turning most traditional constraints into new objective functions. This seems more natural for real world problems where choosing a threshold may seem arbitrary [5]. As a result, a wide set of optimal solutions (Pareto set) may be found. Therefore, an engineer may have a whole set of optimal alternatives before deciding which solution is the best compromise of different (and sometimes contradictory) features. This approach has already been treated as a *Multi-objective Optimization Problem* (MOP) with two conflicting objective functions [6].

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To solve the Reactive Power Compensation Problem, this paper presents a new approach based on the *Strength Pareto Evolutionary Algorithm* (SPEA) [7], which is a MOEA with an external population of Pareto Optimal solutions that best conform a Pareto Front, provided by a clustering process that saves the most representative solutions.

II. Mathematical formulation

For the purposes of this paper, the following assumptions where considered in the formulation of the problem:

- shunt-capacitor/reactor bank cost per MVAr is the same for all busbars of the power system;
- power system is considered only at peak load.

Based on these considerations, four objective functions F_i (to be minimized) have been identified [4, 8]: F_1 and F_2 are related to investment and transmission losses, while F_3 and F_4 are related to quality of service. The objective functions to be considered are:

F_1: *Investment in reactive compensation devices*

$$F_1 = \sum_{i=1}^{n} k\left|B_i\right| \tag{1}$$

$$k = \begin{cases} \alpha & \text{if} \quad 0 \le B_i < B_m \\ \beta & \text{if} \quad -B_m < B_i < 0 \end{cases}$$

s.t.: $F_1 \le F_{1m}$;

where: F_1 is the total required investment; F_{1m} is the maximum amount available for investment; B_i is the compensation at busbar i measured in MVAr; B_m is the absolute value of the maximum amount of compensation in MVAr allowed at a single busbar of the system; α is the cost per MVAr of a capacitor bank; β is the cost per MVAr of a reactor bank and n is the number of busbars in the electric power system.

F_2: *Active power losses*

$$F_2 = P_g - P_l \ge 0 \tag{2}$$

where: F_2 is the total transmission active losses of the power system in MW; P_g is the total active power generated in MW and P_l is the total load of the system in MW.

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F_3: *Average voltage deviation*

$$F_3 = \frac{\sum_{i=1}^{n} \left| V_i - V_i^* \right|}{n} \tag{3}$$

where: F_3 is the per unit (pu) average voltage difference; V_i is the actual voltage at busbar i (pu) and V_i* is the desired voltage at busbar i (pu).

F_4: *Maximum voltage deviation*

$$F_4 = \max_i \left(V_i - V_i^* \right) = \left\| \mathbf{V} - \mathbf{V}^* \right\|_\infty \geq 0 \tag{4}$$

where F_4 is the maximum voltage deviation from the desired value (pu); $\mathbf{V} \in \Re^n$ is the voltage vector (unknown) and $\mathbf{V}^* \in \Re^n$ is the desired voltage vector.

In summary, the optimization problem to be solved is the following:

$$\min \ \mathbf{F} = \begin{bmatrix} F_1 & F_2 & F_3 & F_4 \end{bmatrix} \tag{5}$$

where

$$\mathbf{F} = \begin{bmatrix} \sum_{i=1}^{n} B_i & P_g - P_l & \dfrac{\sum_{i=1}^{n} \left| V_i - V_i^* \right|}{n} & \left\| \mathbf{V} - \mathbf{V}^* \right\|_\infty \end{bmatrix}$$

is known as *objective vector*,

subject to $F_1 \leq F_{1m}$ and the load flow equations [9]:

$$\begin{aligned} P_k &= V_k \sum_{i=1}^{n} Y_{ki} V_i \cos\left(\delta_k - \delta_i - \theta_{ki}\right) \\ Q_k &= V_k \sum_{i=1}^{n} Y_{ki} V_i \sin\left(\delta_k - \delta_i - \theta_{ki}\right) \end{aligned} \tag{6}$$

where: V_k is the voltage magnitude at node k; Y_{ki} is the admitance matrix entry corresponding to nodes k and i; δ_k is the voltage phase angle at node k; θ_{ki} is the phase admitance matrix entry corresponding to nodes k and i; P_k is the active power injected at node k; Q_k is the reactive power injected at node k.

To represent the amount of reactive compensation to be allocated at each busbar i, an unknown vector **B**, known as *decision vector* [7], is used to indicate the size of each reactive bank in the power system, i.e.:

$$\mathbf{B} = \begin{bmatrix} B_1 & B_2 & \cdots & B_n \end{bmatrix}, B_i \in \Re, \left| B_i \right| \leq B_m \tag{7}$$

The set of solutions of a multi-objective optimization problem consists of all decision vectors **B** for which the corresponding objective vectors **F** can not be improved in any dimension without degradation in another. This set of decision vectors are known as *Pareto Optimal*, represented as *P*. The corresponding set of objective vectors **F** calculated using equations (1) to (4) conform a set known as *Optimal Pareto Front*, denoted *PF* [7].

Because the *true* Pareto Optimal Set (termed P_{true}), with its corresponding PF_{true}, are not completely known in practice without extensive calculation (computationally not feasible in most situations), it would be normally enough for practical purposes to find a *known* Pareto Optimal Set, termed P_{known}, with its corresponding Pareto Front PF_{known}, close enough to the true optimal solution [5].

III. PROPOSED METHOD

A new approach based on the Strength Pareto Evolutionary Algorithm was developed for this work. This method, closely related to Genetic Algorithms [10], is based on generating a stored *External Population* composed by the best known individuals **B** of a general evolutionary population. This external group of solutions conforms P_{known}, available at each moment of the computation, i.e., the best known approximation to P_{true}. The original SPEA evaluates an individual's fitness depending on the number of decision vectors it dominates in an evolutionary population, i.e., decision vectors that are not better in any objective function F_i, but with a worse objective function F_i for at least one value of i.

SPEA preserves population diversity using Pareto dominance relationship and incorporating a clustering procedure in order to reduce the nondominated set without destroying its characteristics. In general, cluster analysis partitions a collection of m elements into g groups of relatively homogeneous elements, where $g < m$, selecting a representative individual for each of the g clusters. That way, a fixed number of g individuals may be maintained in the external population preserving the main characteristics of the Pareto Front [7].

An important issue with SPEA is its converge property, assured by *Theorem 4* proved in [5], a characteristic not always present in other MOEAs. Consequently, the algorithm implemented for this work is based on the original SPEA [7], but differs from it in the following aspects:

- *Heuristic Initialization.* A special heuristic method is used to generate the initial population in order to obtain individuals electrically well compensated. The proposed heuristic is based on encouraging compensation at busbars with large number of branches and voltage profile far from the desired value. This is done by using a method summarized as follows:
  a. Choose a total amount of compensation B_{tot}.
  b. For each busbar i of the system, calculate a factor K_i

using the following expression:

$$K_i = \begin{cases} \left(V_i - V_i^*\right) l_i & \text{if } V_i < V_i^* \\ 0 & \text{if } V_i \geq V_i^* \end{cases}$$

where l_i is the number of branches connected to node i. $K_i = 0$ indicates that no reactive compensation is heuristically assigned to busbar i.

c. Normalize K_i using:

$$K_i' = \frac{K_i}{\sum_{i=1}^{n} K_i}$$

d. Compensate each busbar i with B_i calculated as follows:

$$B_i = K_i' B_{tot}$$

- *Local Optimization.* A special heuristic technique is implemented to improve individuals based on determining an adequate search direction using the power flow mismatch expression [4]:

$$\begin{bmatrix} \mathbf{\Delta P} \\ \mathbf{\Delta Q} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_1 \mathbf{J}_2 \\ \mathbf{J}_3 \mathbf{J}_4 \end{bmatrix} \begin{bmatrix} \mathbf{\Delta \delta} \\ \mathbf{\Delta V} \end{bmatrix} \tag{8}$$

From (8) and neglecting $\mathbf{J}_3$ as well as the non-diagonal elements of $\mathbf{J}_4 = \{J_{4_{ij}}\}$, the following expression is derived:

$$\Delta Q_i \approx J_{4_{ii}} \Delta V_i = J_{4_{ii}} \left(V_i^* - V_i\right) \tag{9}$$

where ΔQ_i is the amount of reactive compensation to be added at busbar i.

- *Stop criterion.* Computation is halted when no new nondominated solution is found to dominate an individual of the external population for a given number N_{stop} of successive generations.

- *Two External Populations*. If only one external population is used, it is possible:

a. to save all found Pareto solutions, but this population may become too large and the evolutionary population looses genetic importance in the search process; or

b. to loose found solutions using clustering to maintain a given number g of external solutions (original SPEA approach).

In this new proposal, two external populations are stored, one with all found nondominated solutions and another with a maximum number g of nondominated individuals, fixed by clustering, that participates in the ordinary evolutionary process. That way, the external population used in the evolutionary process does not diminish the influence of the evolutionary population and no optimal solution is lost. Note that this second external population may be stored on disk, because it does not participate in the evolutionary process.

- *Freezing.* Inspired in Simulated Annealing technique, probabilities (of mutation P_m, crossover P_c and for using the local optimization P_{lo}) change with the number of generations and fitness value, freezing at the end of the computation to improve convergence [11].

The proposed method may be summarized as follows:

1. Generate an initial population *Pop* using the heuristic method previously exposed and create two empty external nondominated sets P_{known} and SP_{known} (*stored external population*).
2. Copy nondominated members of *Pop* to P_{known} and SP_{known}.
3. Remove individuals within SP_{known} which are covered (dominated) by any member of SP_{known}.
4. Remove solutions within P_{known} which are covered by any member of SP_{known}.
5. If the number of externally nondominated solutions in P_{known} exceeds a given maximum g, clustering is applied in order to reduce the external population to a size g.
6. Calculate the fitness of each individual in *Pop* as well as in P_{known} using standard SPEA fitness assignment procedure.
7. Select individual from *Pop* + P_{known} (multiset union) until the mating pool is filled. In this study, roulette wheel selection is used.
8. Apply P_{lo}, P_c and P_m to determine whether and individual is locally optimized or selected for crossover and mutation, in which case, standard genetic operators are applied.
9. Go to step 2 if stop criterion is not verified.

IV. EXPERIMENTAL ENVIRONMENT

As a study case, the IEEE 118 Bus Power Flow Test Case has been selected [12]. In order to stress the original system, its active and reactive loads were incremented by 40%, turning the power network in an adequate candidate for reactive power compensation.

For comparison purposes, the Pareto set generated by the proposed method has been compared to Pareto sets obtained using four different methods:

1· Compensation schemes elaborated by a team of specialized engineers using standard computational programs (*Specialist*).

2· Original SPEA implementation, with random initial population [7] (*SPEA*).

3· Original SPEA implementation with heuristic initialization ($SPEA^{+}$).

4· A special SPEA with heuristic initialization and the mutation genetic operator replaced by the previous exposed local optimization ($SPEA^{lo}$).

For the experimental results presented in the following section, it has been assumed that $\alpha = \beta$, i.e., capacitor and reactor banks have the same cost per MVAr. At the same time, N_{stop} = 100 was experimentally chosen.

To evaluate the experimental results using all five methods, an appropriate test suite metrics is used [5], because no single metric can entirely capture total MOEA performance, effectiveness and efficiency. The test suit comprises the following metrics:

1) *Overall Nondominated Vector Generation (N)*

$$N \stackrel{\Delta}{=} \left|PF_{known}\right|_c$$

where $\left|\cdot\right|_c$ denotes cardinality.

This metric indicates the number of solutions in PF_{known}. A good PF_{known} set is expected to have a large number of individuals, in order to offer a wide variety of options to the engineer.

2) *Overall Nondominated Vector Generation Ratio* (*ONVGR*)

$$ONVGR \stackrel{\Delta}{=} \frac{N}{\left|PF_{true}\right|_c}$$

It denotes the ratio between the number of solutions in PF_{known} to the number of solutions in PF_{true}. Since the objective is to obtain a PF_{known} set as similar as possible to PF_{true}, a value near to 1 is desired.

3) *Error Ratio* (*E*)

$$E \stackrel{\Delta}{=} \frac{\sum_{i=1}^{N} e_i}{N};$$

$$e_i = \begin{cases} 0 & \text{if a vector in } PF_{known} \text{ is also in } PF_{true} \\ 1 & \text{otherwise} \end{cases}$$

This ratio reports the proportion of objective vectors in PF_{known} that are not members of PF_{true}. Therefore; an Error Ratio *E* close to 1 indicates a poor correspondence between PF_{known} and PF_{true}, i.e., *E* = 0 is desired.

4) *Generational Distance (G)*

$$G \stackrel{\Delta}{=} \frac{\left(\sum_{i=1}^{N} d_i^2\right)^{1/2}}{N}$$

where d_i is the Euclidean distance (in objective space) between each objective vector **F** in PF_{known} and its nearest member in PF_{true}. A large value of *G* indicates PF_{known} is far from PF_{true}, being *G* = 0 the ideal situation.

5) *Maximum Pareto Front Error* (*ME*)

$$ME \stackrel{\Delta}{=} \max_j \left(\min_i \left\| \mathbf{F}^i - \mathbf{F}^j \right\|_\infty \right)$$

$$\mathbf{F}^i \in PF_{true} \ ; \ \mathbf{F}^j \in PF_{known}$$

It indicates the maximum error band that, when considered with respect to PF_{known}, encompasses every vector in PF_{true}. Ideally, *ME* = 0 is desired.

Since most of these metrics reflect the likeness between the true Pareto Front Optimal set PF_{true} and a computed Pareto Front set PF_{known}, a good approximation of PF_{true} is built by gathering all nondominated individuals from all five sets. In other words, for the following results, PF_{true} is approximated by the best known solutions of all our experiments.

V. Experimental results

Tables I and II present experimental results using the IEEE-118 study case, showing the figures obtained by all five methods. For the methods used to compare the proposed approach, the best results obtained by a single run have been selected, having the SPEA implementations reached a stagnant population, i.e., no new solutions are obtained with new generations for N_{stop} = 100 generations. On the other hand, the proposed method has been stopped using a maximum number of generation criterion, since it continues generating new solutions reaching more than 2000 stored solutions (SP_{known}). This is an important advantage since it gives the user a wider variety of alternative solutions.

TABLE I
EXPERIMENTAL RESULTS: 60 GENERATION RUN OF THE PROPOSED METHOD

Metrics	*Specia-list*	*SPEA*	$SPEA^{+}$	$SPEA^{lo}$	*Proposed Method*
N	170	100	150	172	181
ONVGR	0.4315	0.2538	0.3807	0.4365	0.4594
E	0.2353	0.9500	0.9533	0.2558	0.2431
G	0.5702	0.7325	0.6060	0.5814	0.5431
ME	0.0852	0.1040	0.2332	0.0324	0.0554

Table I presents experimental results running only 60 generation of our proposed approach while the other methods run until convergence. Table II presents figures when the proposed method run for 200 generations. In both tables, best figures are indicated by a shadowed cell.

TABLE II
EXPERIMENTAL RESULTS: 200 GENERATION RUN OF THE PROPOSED METHOD

Metrics	*Specia-list*	*SPEA*	$SPEA^{+}$	$SPEA^{lo}$	*Proposed Method*
N	170	100	150	172	222
ONVGR	0.4208	0.2475	0.3713	0.4257	0.5495
E	0.3235	0.9800	0.9867	0.4128	0.1486
G	0.6138	0.7635	0.6221	0.6022	0.5315
ME	0.0852	0.0948	0.2294	0.0462	0.0342

For the first two metrics *N* and *ONVGR*, it is clear that the proposed method has the best performace, since it generates the widest variety of solutions. This fact is even more evident when the stored external population is considered.

In Table I, the compensation set elaborated by the specialists has the smallest Error Ratio *E*, closely followed by the proposed method, what gives an idea of the good job being done by the engineers; however, with a greater number of generations, the proposed method improves sensitively its performance, reducing in nearly 40% its error ratio, outperforming other methods, as shown in Table II.

The values obtained for the Generational Distance *G* show that the proposed method has the best results in both tables, offering a PF_{known} set closer to PF_{true}.

Table I indicates that the proposed method occupies the second place in the Maximum Pareto Front Error (*ME*) ranking for a lower number of generations, beaten by the $SPEA^{lo}$ implementation. However, with a larger number of generations, the proposed method have the smallest *ME* value (see Table II).

A final fact to be emphasized is that the proposed method improves its performance for most metrics with a larger number of generations, while the other methods have converged before whole exploration of search space, what is known as premature convergence.

VI. CONCLUDING REMARKS

In this paper, Reactive Compensation Problem is first treated as a *Multi-objective Optimization Problem* with 4 conflicting objective functions: (i) investment in reactive compensation devices, (ii) active power losses, (iii) average voltage deviation and (iv) maximum voltage deviation.

To solve the problem, a new approach based on *SPEA* is proposed. This new approach introduces several proposals as: (i) heuristic initialization, (ii) a local optimization technique, (iii) a stop criterion, (iv) two external populations and (v) a freezing feature.

For comparison purposes, the solution set obtained in a single run of the proposed method is compared to four sets of solutions calculated as the best of several runs using other methods or as the best set of solutions calculated by a team of specialists.

Experimental results using the proposed approach demonstrated several advantages when using the proposed method, such as a set of solutions closer to the *True Pareto Set* outperforming other methods in every studied figure of merits, and a wider variety of options. This last feature is of special importance, since a richer set of alternatives are offered to the network planners. In order to select sub-sets of solutions which best fit the interests of the user, an adaptive constrain philosophy is suggested. That way, the network engineer may restrict the constraints to reduce the number of solutions after having a good idea of the whole Pareto solutions, searching forward only in the redefined domain. This process may continue iteratively until a good solution with an acceptable compromise among objective functions is found.

As future work, new specialized genetic operators are being developed to locally improve reactive compensation of a given individual. At the same time, other objective functions (such as voltage stability margin) are going to be considered. Finally, parallel asynchronous computation using a network of computers are considered for larger networks with more objective functions.

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